Protograph LDPC Code Extension for Key Reconciliation of Satellite-based Optical Key Distribution Systems

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Febuary 27th, 2024

Outline

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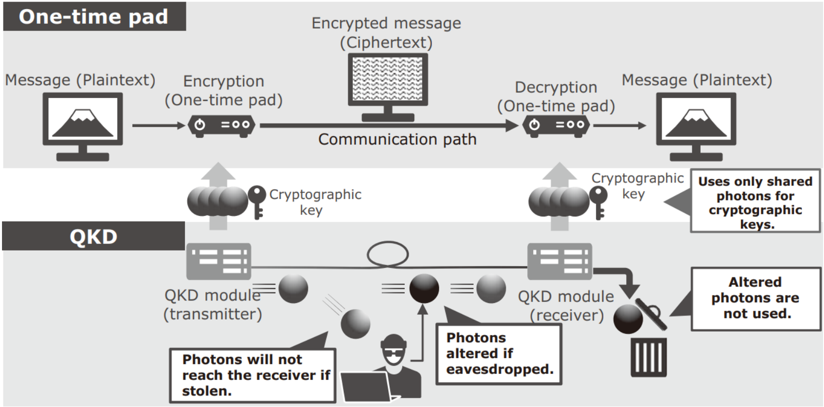
I. Introduction

II. Proposed Blind Reconciliation

III. Simulation Results

Quantum Key Distribution (QKD)

Quantum key distribution (QKD): *a key distribution protocol based on quantum mechanics*



Free Space Optical (FSO)-based Satellite QKD Systems

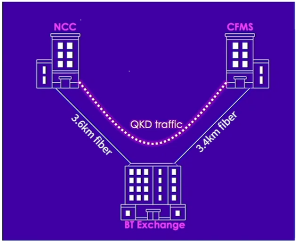


Figure: Optical fiber QKD systems.

- Have been widely commercialized
- Can not support mobile users

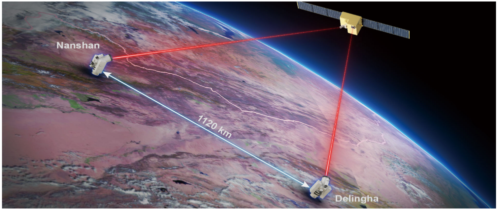


Figure: *Micius*, the world's first quantum satellite experiment

- Can support mobile users via the FSO channel
- Provide global coverage using satellites

FSO-based satellite QKD systems are potential approaches for future mobile networks.

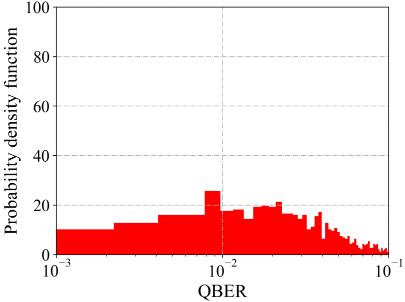
Challenging Issues: Uncertainty Channel

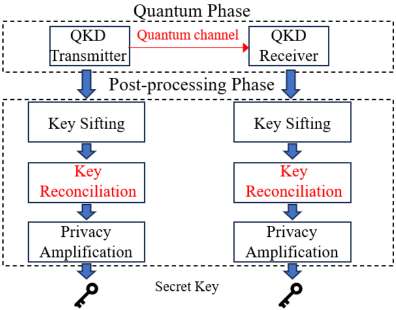
Atmospheric Turbulence:

- **Cause:** Inhomogeneity in refractive-index along the propagation path of the optical signal
- Lead to **fluctuating quantum bit-error rate (QBER)**

In general, QKD protocols always include a step in the post-processing phase to correct errors, namely **key reconciliation**.

$\Longrightarrow$ *It is necessary to have a proper design of key reconciliation for satellites QKD systems.*





Key Reconciliation based on Error Correction Code

- **Key reconciliation:** Both users (Alice and Bob) try to correct the errors in their keys while minimizing the information leakage
- One of the main approaches is using the syndrome-based error correction codes
- **Low-density parity-check (LDPC) code** is widely considered thanks to its capacity-approaching performance and low-decoding complexity

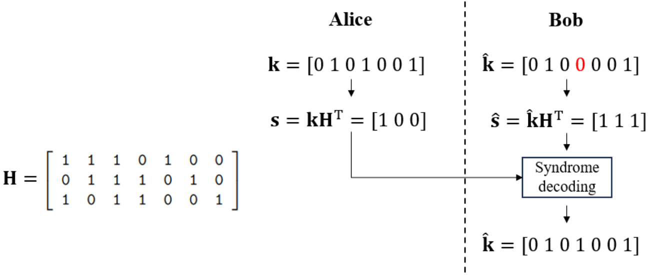


Figure: An example of syndrome decoding with linear block code

Existing Approaches

There are three main approaches

1. **Fixed-rate Reconciliation:** A fixed code rate is used to reconcile for all blocks
2. **Adaptive-rate Reconciliation:** Based on estimated QBER, choose the best code rates among a set of code rates to reconcile
   - To estimate the QBER, Alice and Bob will reveal a portion of sifted keys (10-25%)
   - If the reconciliation fails, both sides discard their sifted keys.

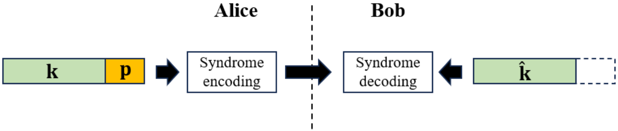
   $\Longrightarrow$ **Fixed-rate** and **adaptive-rate** may be *inefficient over turbulence FSO channels.*
3. **Blind Reconciliation:** If the reconciliation fails, incremental information will be sent to help decoding
   - Blind reconciliation was first proposed in [1] and has been investigated in several studies [2]–[5].

➡ *Blind reconciliation is a potential approach for key reconciliation of satellite-based QKD systems.*

Blind Reconciliation based on Puncturing LDPC Codes

Key Idea: Alice adds a certain number of random bits to her key before syndrome encoding

- These bits are unknown to Bob, and he treats them as *punctured bits* when decoding
- Alice adjusts the code rates by revealing the values of these punctured bits.



The range of code rates depends on the *the percentage of added random bits*, α

$$R_{\text{max}} = \frac{R_{\text{base}}}{1-\alpha} \geq R \geq \frac{R_{\text{base}} - \alpha}{1-\alpha} = R_{\text{min}}$$

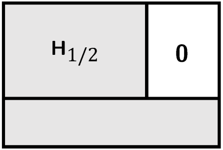
Limited code rate range $\Longrightarrow$ Inefficient for the considered systems

Motivations & Contributions

A viable solution to construct a rate-compatible (RC)-LDPC code family is **code extension** method

- A new parity check matrix is obtained by extending another parity check matrix.

$\mathbf{H}_{1/3} =$



Contribution:

We propose a design of blind reconciliation based on the RC-LDPC code extension.

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Proposed Structure

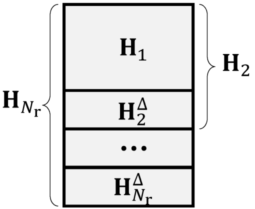


Figure: Proposed structure

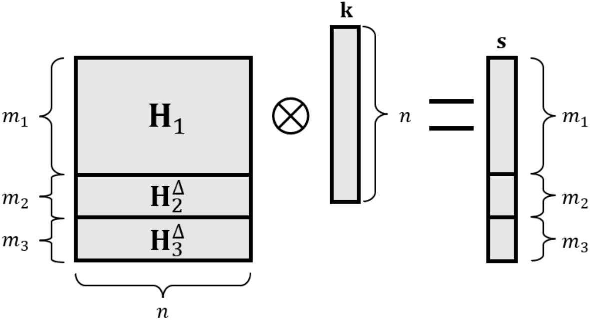
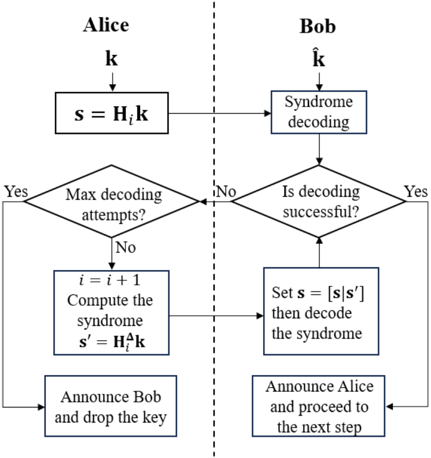


Figure: An example of nested syndrome with the proposed structure.

Flowchart of Proposed Blind Reconciliation

Initalization: Set $i = 1$



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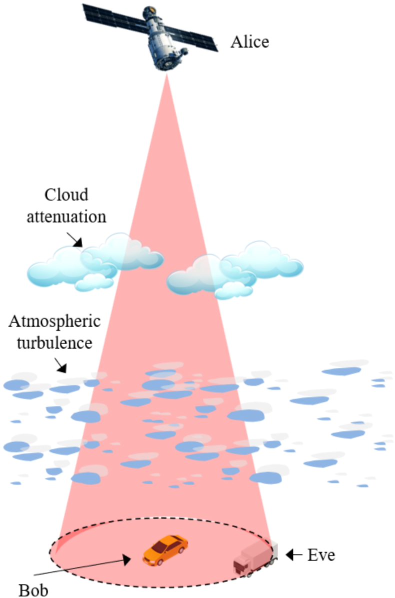
Considered System Model



System model:

- An LEO satellite (Alice) distributes key materials to a ground vehicle (Bob)
- Dual-threshold/direct detection key distribution is used

FSO Channel Model:

- Atmospheric Turbulence
- Cloud Attenuation
- Beam-spreading loss

An adversary's car (Eve) follows Bob and eavesdrops on the beam footprint.

Final Key Rate

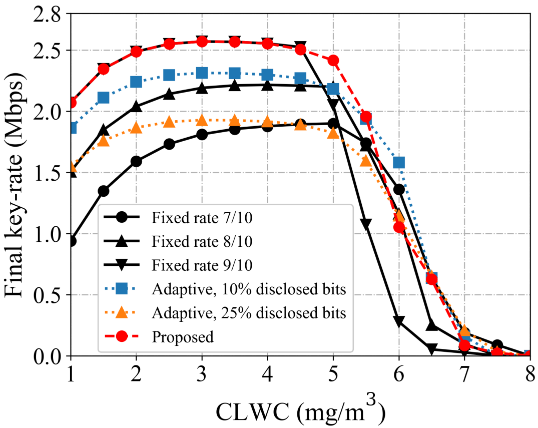
The final key rate is calculated as

$$\mathsf{KR} = \sum_{i=1}^{N_\mathsf{r}} \underbrace{\left(1 - \overline{\mathsf{FER}}_i\right)}_{\substack{\text{Prob. of} \\ \text{successful} \\ \text{reconciliation}}} \underbrace{\left(R_i - I_{AE}\right)}_{\substack{\text{Information after} \\ \text{privacy amplification}}} \underbrace{\frac{N}{R_\mathsf{b} P_\mathsf{sift}}}_{\substack{\text{Average} \\ \text{block rate}}}$$

where

- I_{AE}: mutual information between the sifted key of Alice and the information obtained by Eve
- N: block length
- R_b: the satellite's data rate
- P_sift: the sift probability.

Comparison among Other Reconcilition Methods



- Consider code rate range $\left(\frac{9}{10}, \frac{8}{10}, \frac{7}{10}\right)$
- Fixed-rate and adaptive-rate consider perfect code rate

➡ The proposal design outperforms the other methods in most of the considered range.

Thank you for your attention!

References

[1] J. Martinez-Mateo, D. Elkouss, and V. Martin, "Blind reconciliation," (2013), [Online]. Available: https://arxiv.org/abs/1205.5729.

[2] E. O. Kiktenko, A. S. Trushechkin, C. C. W. Lim, Y. V. Kurochkin, and A. K. Fedorov, "Symmetric blind information reconciliation for quantum key distribution," *Phys. Rev. Appl.*, vol. 8, no. 4, p. 044 017, 2017.

[3] Z. Liu, Z. Wu, and A. Huang, "Blind information reconciliation with variable step sizes for quantum key distribution," *Scientific Reports*, vol. 10, no. 1, p. 171, 2020.

[4] E. O. Kiktenko, A. O. Malyshev, and A. K. Fedorov, "Blind information reconciliation with polar codes for quantum key distribution," *IEEE Commun. Lett.*, vol. 25, no. 1, pp. 79–83, 2021.

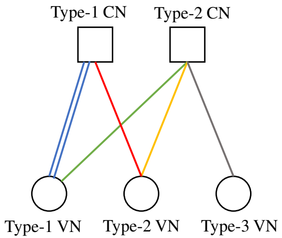
[5] H.-K. Mao, Y.-C. Qiao, and Q. Li, "High-efficient syndrome-based ldpc reconciliation for quantum key distribution," *Entropy*, vol. 23, no. 11, p. 1440, 2021.

Appendix: Protograph

- **Protograph** is a *small Tanner graph* serving as a template to construct the LDPC.
- **The structured LDPC codes** *inherits* from the protograph
  - Code rate & distributions of degree of nodes

  $\Longrightarrow$ The LDPC codes can be faster optimized by optimizing on the protograph level
- A protograph can be equivalently represented by a **base matrix**

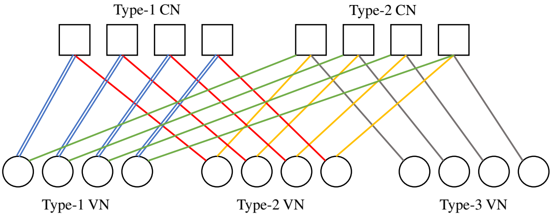
$$\mathbf{B} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$



Copy-and-Permute (1)

- The LDPCC with desired information length is derived from the protograph by a **"copy-and-permute" operation**
  - The derived graph is called *lifted graph*
- **First step:** Making T copies of the protograph

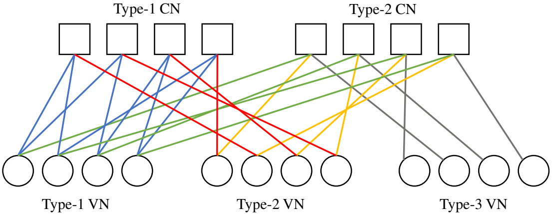
$$\mathbf{B}_4 = \begin{pmatrix} 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$



Copy-and-Permute (2)

- **Second step:** Permuting the end-points of each edge between nodes of the same type

$$\mathbf{B}_4 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$



- Edges in the lifted graph are distributed following the edge types in the protograph

⇒ The lifted graph **inherits** properties of the protograph