Outage Performance of High Altitude Platform-Unmanned Aerial Vehicle Free Space Optical Links

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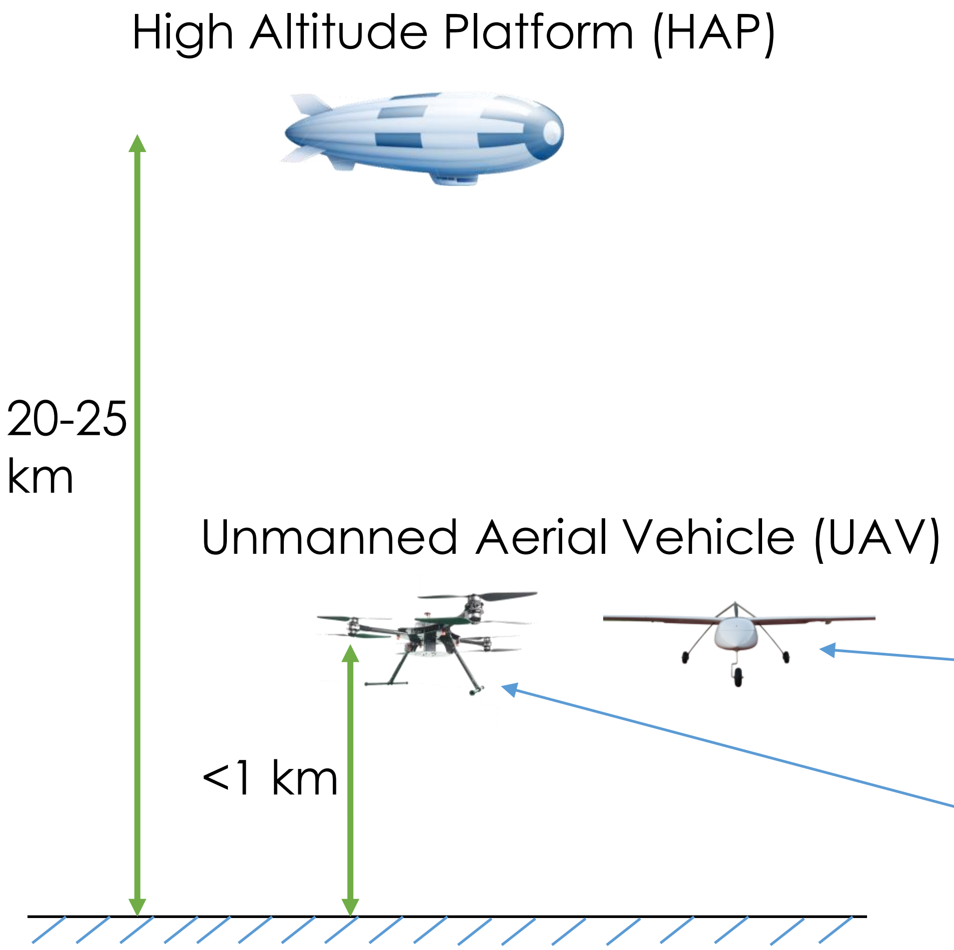
Contents

High Altitude Platform-Unmanned Aerial Vehicle (HAP-UAV)



- High Altitude Platforms can provide wide-scale wireless coverage for large geographic areas
- Unmanned Aerial Vehicle
  - Fixed wing UAV: can not hover (remain in one place in the air)
  - Rotary wing UAV: can hover

Related Works

Papers / Research	[1]	[2]	[3]
Free space optical (FSO) links	Inter-HAP links Inter-UAV links	Inter-HAP links	Inter-UAV links Ground-UAV links UAV-Ground links
Channel Modeling			
Gaussian beam	☐	☐	☒
Atm. Turbulence	☒	☒	☒
Hovering UAV	☐	☐	☒

[1] E. Leitgeb, K. Zettl, S. S. Muhammad, N. Schmitt and W. Rehm, "Investigation in Free Space Optical Communication Links Between Unmanned Aerial Vehicles (UAVs)," *2007 9th International Conference on Transparent Optical Networks*, Rome, 2007, pp. 152-155

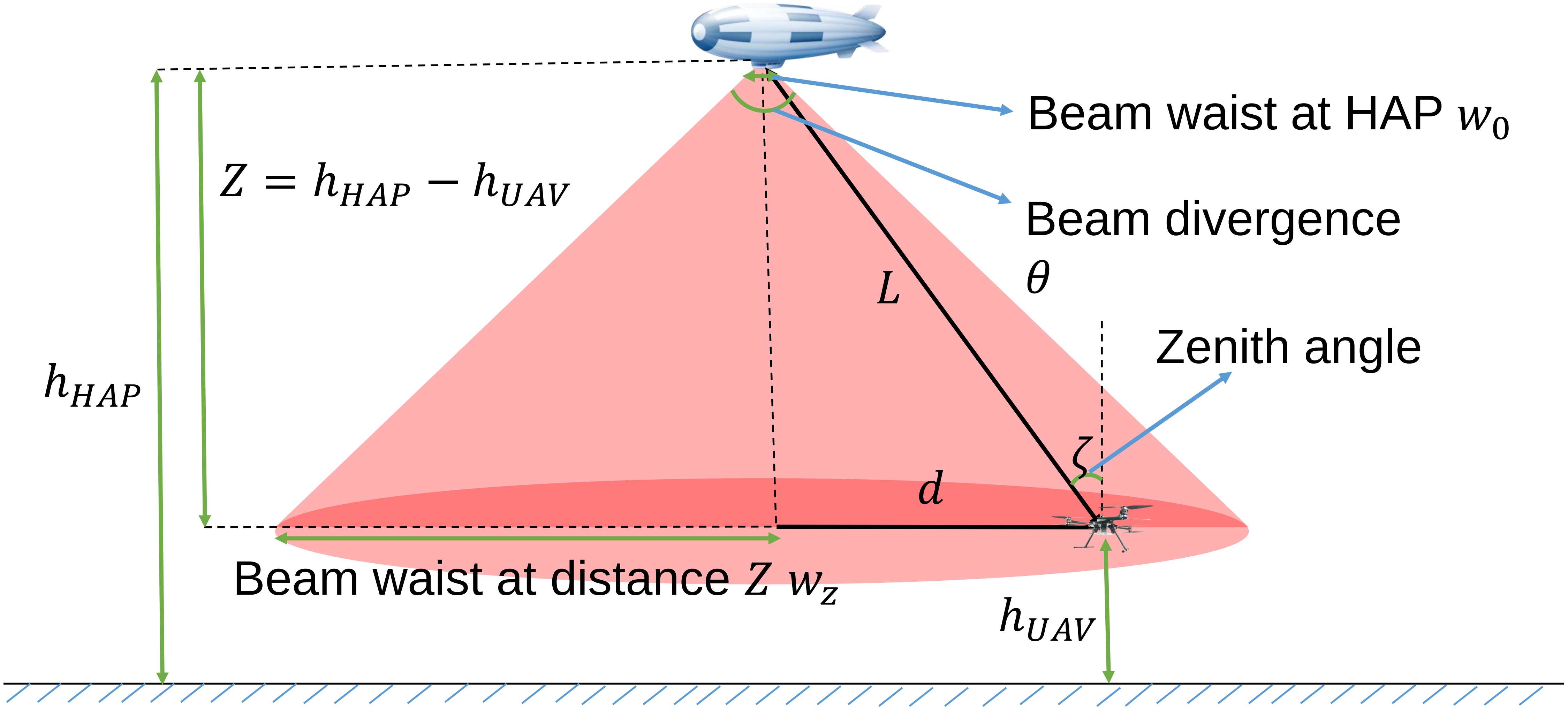
[2] H. Akbar and Iskandar, "Outage probability analysis for optical inter-platform HAPS-Link over log normal turbulence channels," *2015 9th International Conference on Telecommunication Systems Services and Applications (TSSA)*, Bandung, 2015, pp. 1-4.

[3] M. T. Dabiri, S. M. S. Sadough and M. A. Khalighi, "Channel Modeling and Parameter Optimization for Hovering UAV-Based Free-Space Optical Links," in *IEEE Journal on Selected Areas in Communications*, vol. 36, no. 9, pp. 2104-2113, Sept. 2018.

Goals

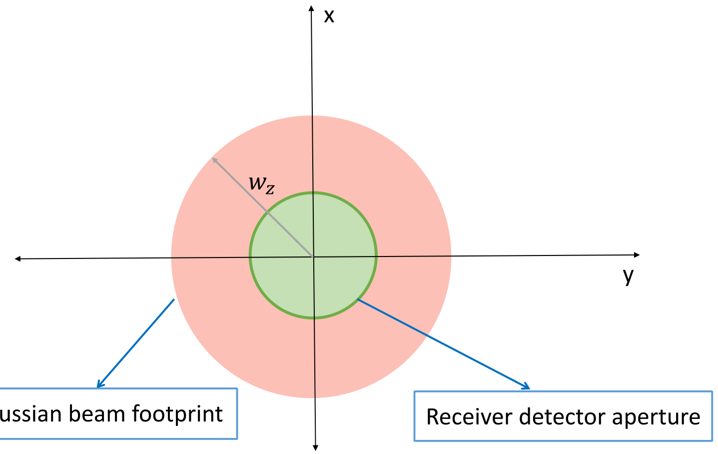
- Derive outage probability and outage capacity of the **HAP-UAV FSO links**
- Take into consideration
  - Gaussian beam
  - Geometric loss and pointing error loss (due to hovering UAV)
  - Atmospheric turbulence

System Model

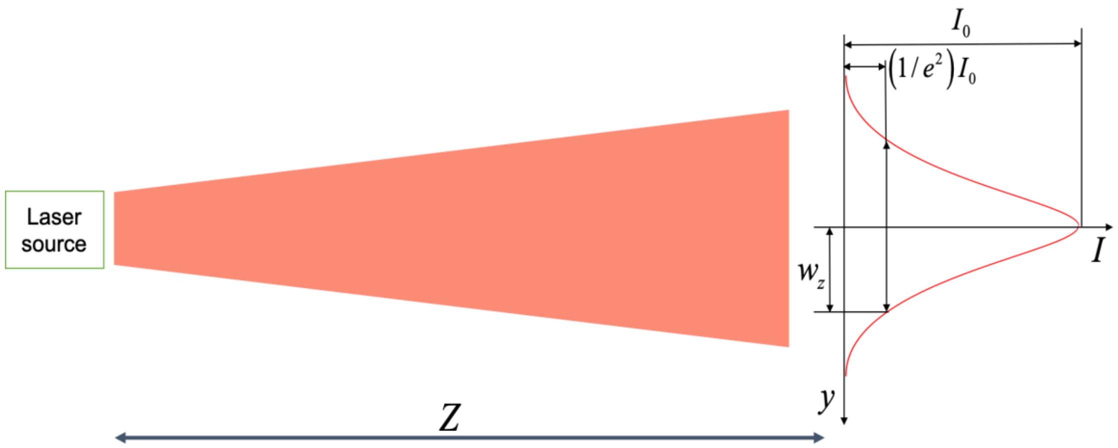


Gaussian beam

The Gaussian beam is considered at the HAP:



The Gaussian beam footprint at the Rx aperture without pointing error.



The Gaussian beam within the transverse plane.

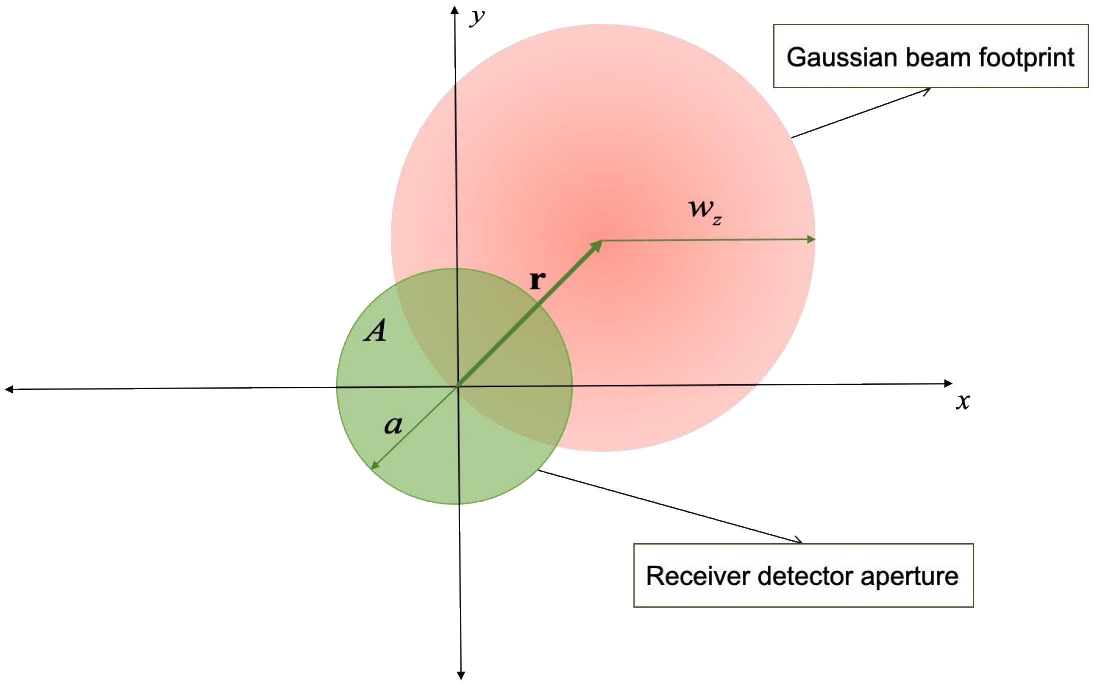
The normalized spatial distribution of the transmitted intensity at distance Z for a Gaussian beam is given as :

$$I_{beam}(\mathbf{p};z) = \frac{2}{\pi w_z^2} \exp\left(\frac{-2\|\mathbf{p}\|^2}{w_z^2} \right)$$

where $\mathbf{p} = [x, y]$ is the radial vector from the beam center, $\|\mathbf{p}\| = \sqrt{x^2 + y^2}$ is the length of $\mathbf{p}$

w_z is the beam waist at distance Z, $w_z = w_0 \left[1 + \left(\frac{Z}{z_0}\right)^2\right]^{1/2}$, w_0 is the beam waist at laser source

Gaussian beam



The misalignment between the Tx and the Rx on the receiver detector plane

With a circular receiver detection aperture of radius a and a Gaussian beam profile I_{beam}, the attenuation because of geometric spread with pointing error $\mathbf{r}$ can be calculated as

$$h_{pe}(\mathbf{r}, z) = \int_A I_{beam}(\mathbf{p} - \mathbf{r}; z)\, d\mathbf{p}$$

h_{pe} is the fraction of the power collected by the detector.

A is the detector area.

The integration can be approximated as the Gaussian form [5]

$$h_{pe}(r_d; z) \approx A_0 \exp\left(-\frac{2r_d^2}{w_{zeq}^2}\right)$$

$r_d = \|\mathbf{r}\|$ is the length of $\mathbf{r}$; $v = \sqrt{\pi}a / \sqrt{2}w_z$

$A_0 = [\mathrm{erf}(v)]^2$ is the fraction of the collected power at $r_d = 0$

$w_{zeq}^2 = w_z^2 \dfrac{\sqrt{\pi}\,\mathrm{erf}(v)}{2v\exp(-v^2)}$ is the equivalent beam width

Channel Model

- The channel coefficient is calculated as

$$h = h_l h_{pe} h_a$$

The atmospheric attenuation (h_l)

It is modeled by the exponential Beer-Lambert Law as

$$h_l = \exp(-\varepsilon L)$$

where ε is the attenuation coefficient, L is the link distance.

The pointing error loss

It is caused by the different locations of the UAVs and the random displacement of the hovering UAVs

The atmospheric turbulence

The probability density function (PDF) of fading coefficient h_a is modeled by a Gamma-Gamma distribution.

The pointing error loss

The scalar component of $\overrightarrow{r_{pe}}$ can be written as

$$x_{\overrightarrow{r_{pe}}} = \|\vec{d}\|\sin(\varphi) + x_{\overrightarrow{r_h}} = d_x + x_{\overrightarrow{r_h}}$$

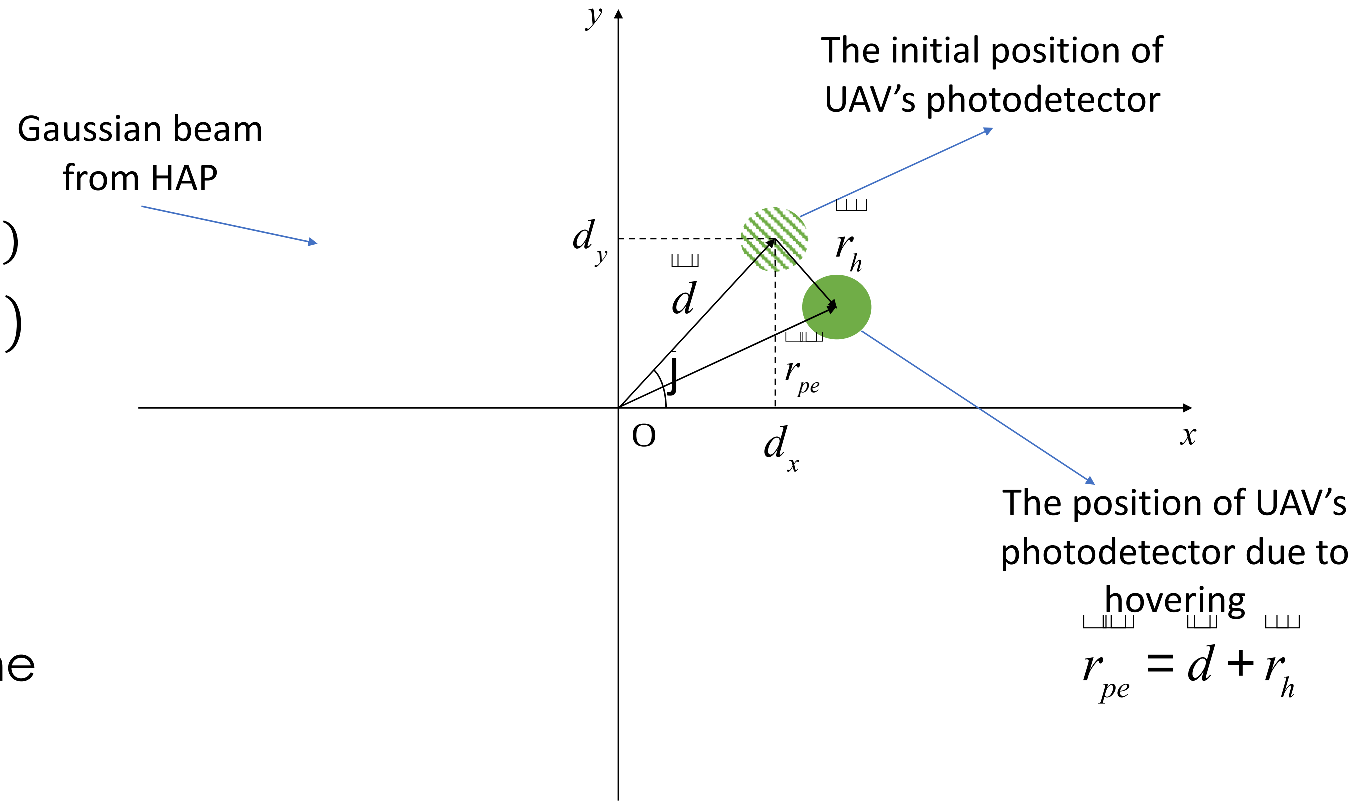
$$y_{\overrightarrow{r_{pe}}} = \|\vec{d}\|\cos(\varphi) + y_{\overrightarrow{r_h}} = d_y + y_{\overrightarrow{r_h}}$$

Because $\left.\begin{matrix} x_{\overrightarrow{r_h}} \sim N(0,\sigma_P^2) \\ y_{\overrightarrow{r_h}} \sim N(0,\sigma_P^2) \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} x_{\overrightarrow{r_{pe}}} \sim N(d_x,\sigma_P^2) \\ y_{\overrightarrow{r_{pe}}} \sim N(d_y,\sigma_P^2) \end{matrix}\right.$

The pointing error loss is expressed as

$$h_{pe} \simeq A_0 \exp\left(-\frac{2r_{pe}^2}{w_{zeq}^2}\right)$$

where $r_{pe} = |\overrightarrow{r_{pe}}| = \sqrt{x_{\overrightarrow{r_{pe}}}^2 + y_{\overrightarrow{r_{pe}}}^2}$, A_0 is the faction of collected power at $r_{pe} = 0$, w_{zeq} is the equivalent beam width.



The PDF of r_{pe} is expressed as $\left.\begin{matrix} x_{\overrightarrow{r_{pe}}} \sim N(d_x,\sigma_P^2) \\ y_{\overrightarrow{r_{pe}}} \sim N(d_y,\sigma_P^2) \end{matrix}\right\} \Rightarrow f_{r_{pe}}(r_{pe}) = \frac{r_{pe}}{\sigma_p^2}\exp\left(-\frac{r_{pe}^2+d^2}{2\sigma_p^2}\right) I_0\left(\frac{r_{pe}d}{\sigma_p^2}\right)$ Rician distribution

The PDF of h_{pe} is expressed as $f_{h_{pe}}(h_{pe}) = \frac{w_{zeq}^2}{4\sigma_p^2 A_0}\left[\exp\left(\frac{-d^2}{2\sigma_p^2}\right)\right]\left(\frac{h_{pe}}{A_0}\right)^{w_{zeq}^2/4\sigma_p^2-1} I_0\left(d\sqrt{-\frac{w_{zeq}^2}{2}\ln\left(\frac{h_{pe}}{A_0}\right)}\Big/\sigma_p^2\right)$

The atmospheric turbulence

The PDF of h_a is given by

$$f_G(h_a) = \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{T(\alpha)\,T(\beta)} h_a^{\frac{\alpha+\beta}{2}-1} k_{\alpha-\beta}\left(2\sqrt{\alpha\beta h_a}\right)$$

where α and β are the effective number of large-scale and small-scale eddies, respectively, and given as

$$\alpha = \left[\exp\left(\frac{0.49\sigma_R^2}{\left(1+1.11\sigma_R^{12/5}\right)^{7/6}}\right)\text{-}1\right]^{-1} \qquad \beta = \left[\exp\left(\frac{0.51\sigma_R^2}{\left(1+0.69\sigma_R^{12/5}\right)^{5/6}}\right)\text{-}1\right]^{-1}$$

where σ_R^2 is the Rytov variance for slant path FSO link and it is expressed as

$$\sigma_R^2 = 2.25k^{7/6}\sec^{11/6}(\zeta)\int_{h_{UAV}}^{h_{HAP}} C_n^2(h)\left(h-h_{UAV}\right)^{5/6}dh$$

where $k = 2\pi/\lambda$, λ is the operational wavelength, ζ is the zenith angle between UAV and HAP, and $C_n^2(h)$ is the refractive index structure parameter. $C_n^2(h)$ can be expressed as

$$C_n^2(h) = 0.00594\left(\frac{w}{27}\right)^2\left(10^{-5}h\right)^{10}\exp\left(-\frac{h}{1000}\right)+2.7\times10^{-16}\exp\left(-\frac{h}{1500}\right)+A_0\exp\left(-\frac{h}{100}\right)$$

where $A_0=1.7\times10^{-14}m^{-2/3}$ is the nominal value of C_n^2 at the ground, w is the wind speed in m/s, and h is the altitude in meters.

The PDF of $h = h_l h_{pe} h_a$ is given by

$$f_h(h) = \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{T(\alpha)T(\beta)} \frac{w_{zeq}^2}{4\sigma_p^2 h_l A_0} \left[\exp\left(\frac{-d^2}{2\sigma_p^2}\right)\right] \left(\frac{h}{h_l A_0}\right)^{\frac{w_{zeq}^2}{4\sigma_p^2}-1} \int_{\frac{h}{h_l A_0}}^{\infty} h_a^{\frac{\alpha+\beta}{2}-1-\frac{w_{zeq}^2}{4\sigma_p^2}} I_0\left(\frac{d\sqrt{-\frac{w_{zeq}^2}{2}\ln\left(\frac{h}{h_l h_a A_0}\right)}}{\sigma_p^2}\right) k_{\alpha-\beta}\left(2\sqrt{\alpha\beta h_a}\right) dh_a$$

The instantaneous electrical signal-to-noise ratio (SNR) is defined as

$$\Upsilon = \frac{R^2 P_t^2 h^2}{\sigma_n^2}$$

where *R* is the receiver's responsivity, P_t is the transmitted power, and σ_p^2 is the noise variance

Then, the PDF of ϒ is expressed as

$$f_\Upsilon(\Upsilon) = \frac{\sigma_n^2}{R^2 P_t^2} \frac{1}{2\sqrt{\frac{\Upsilon\sigma_n^2}{R^2 P_t^2}}} \left[\exp\left(\frac{-d^2}{2\sigma_p^2}\right)\right] \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{T(\alpha)T(\beta)} \frac{w_{zeq}^2}{4\sigma_p^2 h_l A_0} \left(\sqrt{\frac{\Upsilon\sigma_n^2}{R^2 P_t^2}} \frac{1}{h_l A_0}\right)^{\frac{w_{zeq}^2}{4\sigma_p^2}-1} \int_{\frac{\sqrt{\frac{\Upsilon\sigma_n^2}{R^2 P_t^2}}}{h_l A_0}}^{\infty} h_a^{\frac{\alpha+\beta}{2}-1-\frac{w_{zeq}^2}{4\sigma_p^2}} I_0\left(\frac{d\sqrt{-\frac{w_{zeq}^2}{2}\ln\left(\sqrt{\frac{\Upsilon\sigma_n^2}{R^2 P_t^2}} \frac{1}{h_l h_a A_0}\right)}}{\sigma_p^2}\right) k_{\alpha-\beta}\left(2\sqrt{\alpha\beta h_a}\right) dh_a$$

Outage probability & Outage capacity

The outage probability: It is the probability that the instantaneous SNR falls below a specific threshold Υ_{th}

The outage probability is calculated by

$$P_{out} = \int_0^{\Upsilon_{th}} f_{\Upsilon}(\Upsilon)\, d\Upsilon$$

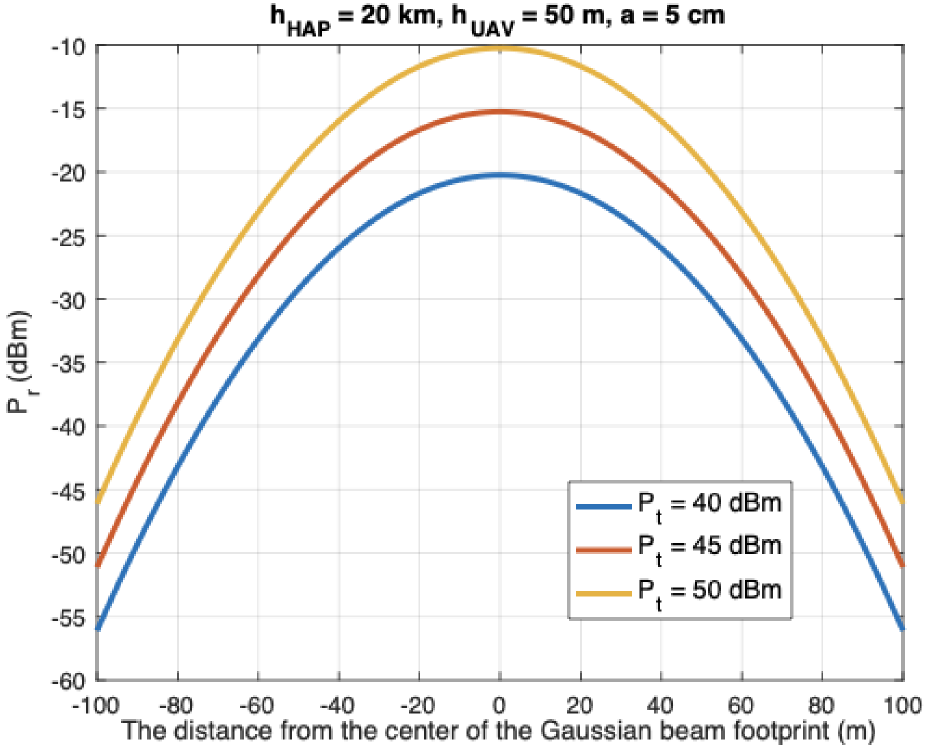
The capacity with outage: is defined as the maximum rate that can be transmitted over a channel with some outage probability. The basis premise is that a high data rate can be sent over the channel and decode correctly except when the channel is in deep fading

Average capacity with outage:

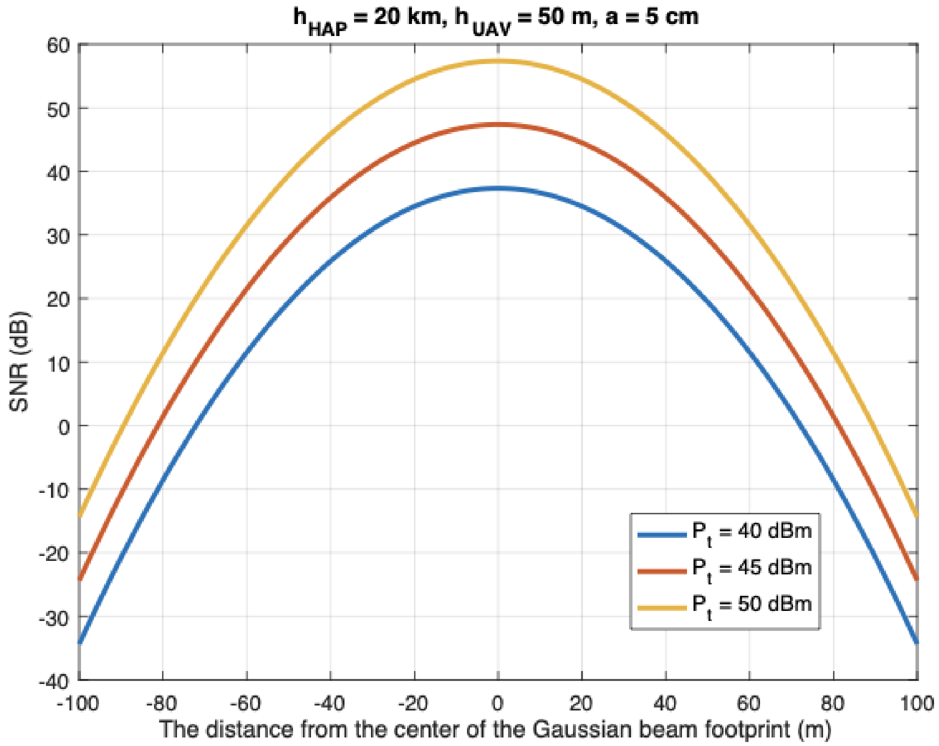
$$C_{out} = (1 - P_{out}) B \log_2(1 + \Upsilon_{th})$$

Numerical and Simulation Results

- The received power versus the distance of the UAV from the center of the Gaussian beam footprint with the different transmitted power



- The signal-to-noise ratio versus the distance from the center of the Gaussian beam footprint with different transmitted power.



Numerical and Simulation Results

HAP

- The wavelength (**λ**): 1550 nm
- The beam waist at HAP (w_0): 0.2 mm
- The height of HAP (h_{HAP}): 20-25 km
- The transmitted power (P_t): 40 - 50 dBm
- The threshold SNR (Υ_{th}): 0 dB
- Bit-rate: 1 Gbps

Channel

- Visibility (V): 30 km (Weather condition: Very clear)

$$\varepsilon = \frac{3.91}{V}\left(\frac{\lambda(nm)}{50}\right)^{\delta}$$

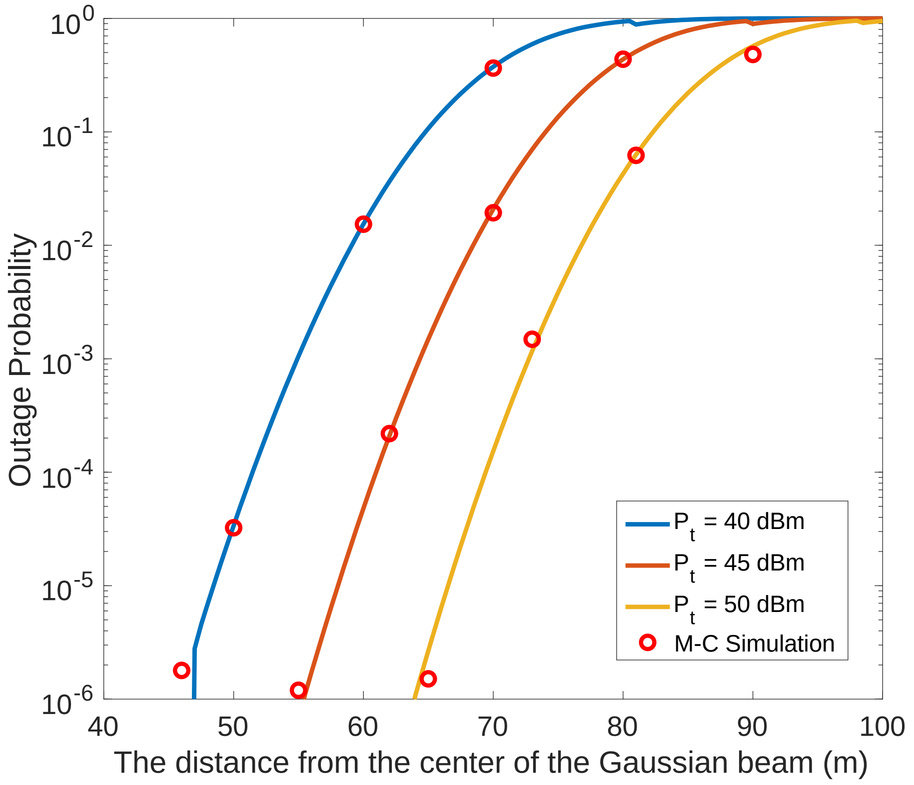
$$\delta = \begin{cases} 1.6 & \text{for V>50 km} \\ 1.3 & \text{for 6 km<V<50 km} \\ 0.585V^{1/3} & \text{for 0 km<V<6 km} \end{cases}$$

(Kruse model)

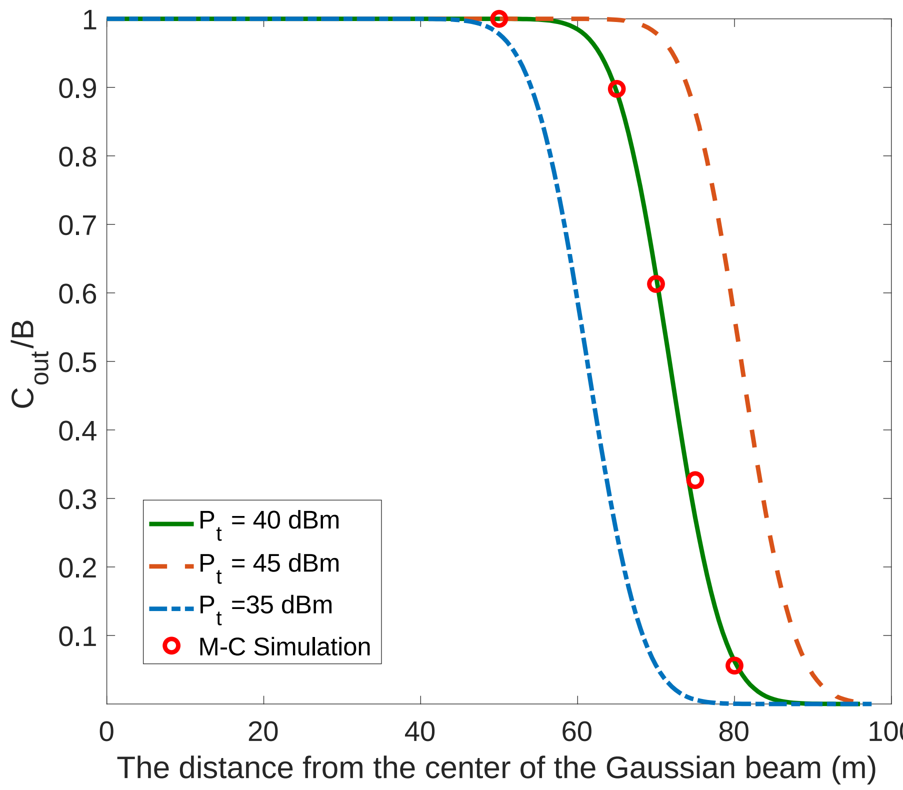
UAV

- The height of UAV (h_{UAV}): 50 m
- The receiver's radius (a): 5 cm
- The responsivity (R) : 0.9 A/W
- The effective noise bandwidth (Δf): 0.5 GHz
- The absolute temperature: 298 K
- The load resistor (R_L): 1 kΩ
- The amplifier noise figure (F_n): 2
- The standard deviation of UAV due to hovering (σ_p): 5m

Numerical and Simulation Results



Outage probability versus the distance from the center of the Gaussian beam footprint

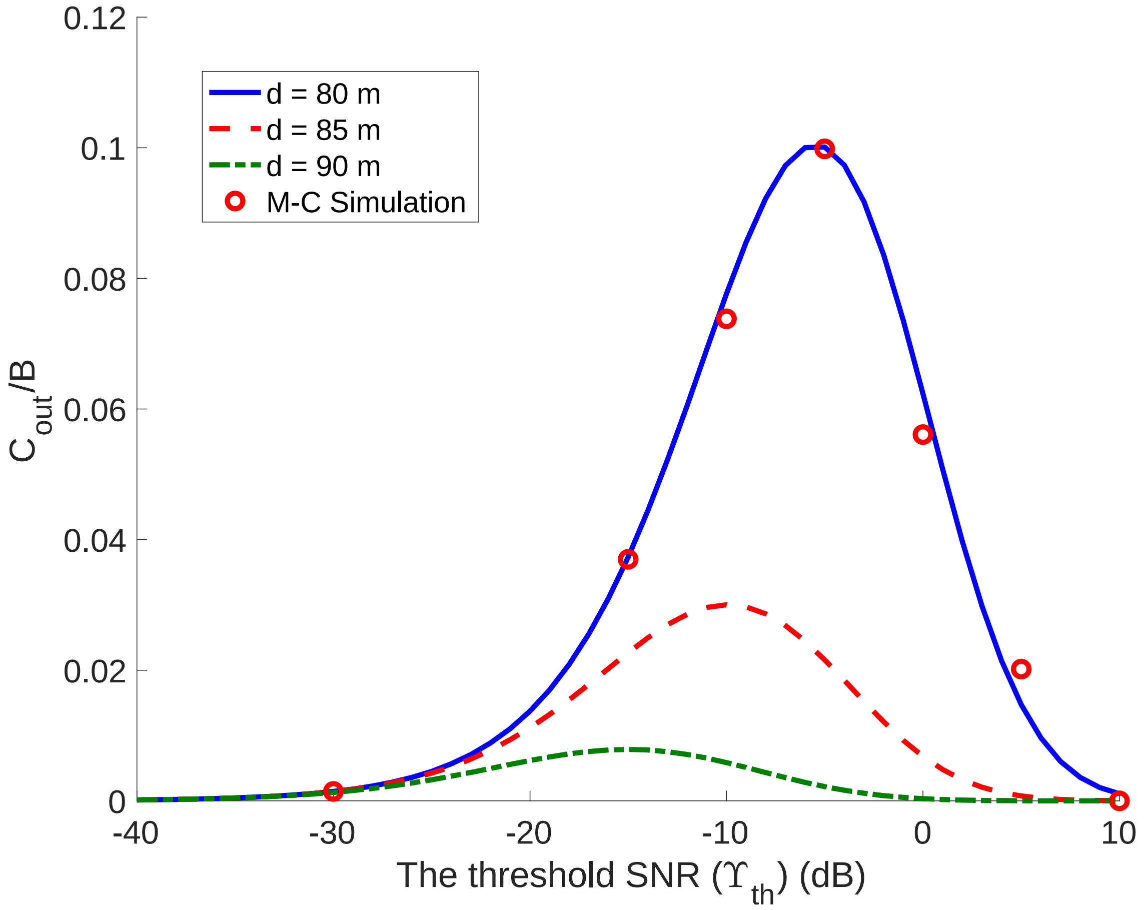


Outage capacity/Bandwidth versus the distance from the center of the Gaussian beam footprint

Numerical and Simulation Results

- Outage capacity/Bandwidth versus the threshold SNR with $P_t = 40\ \mathrm{dBm}$

=> We can find the optimum threshold SNR for each location of UAV



Conclusion

- The channel model for HAP-UAV FSO links under impact of atmospheric attenuation, pointing error loss, and atmospheric turbulence is investigated.
- **Outage probability** versus the distance from the center of the Gaussian beam footprint and **Outage capacity/Bandwidth** versus the distance from the center of the Gaussian beam footprint are derived. These numerical results are verified by Monte-Carlo simulation.
- **Outage capacity/Bandwidth** versus the threshold SNR is also derived to find the optimum threshold SNR for each location of UAVs.

Thank you!