Orthogonal Frequency Division Multiplexing

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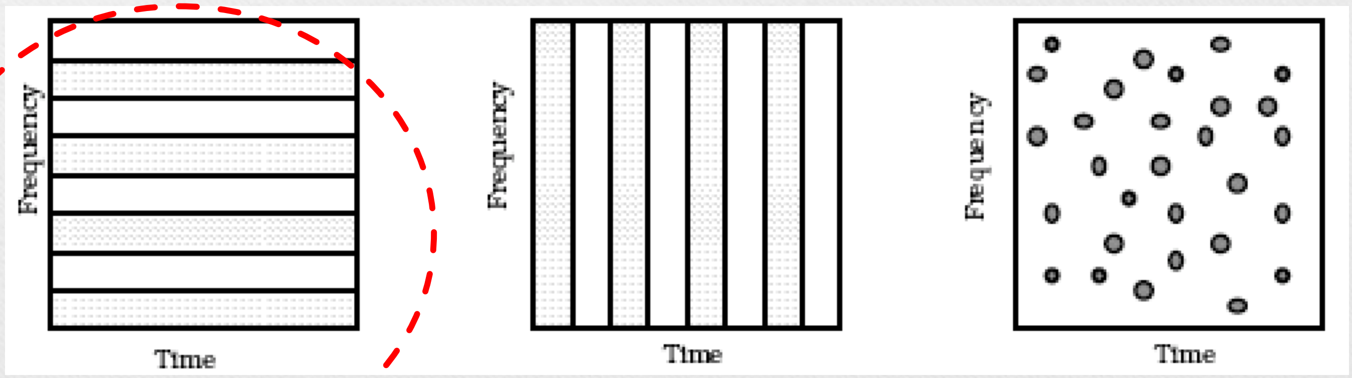
Contents

1. Introduction to OFDM

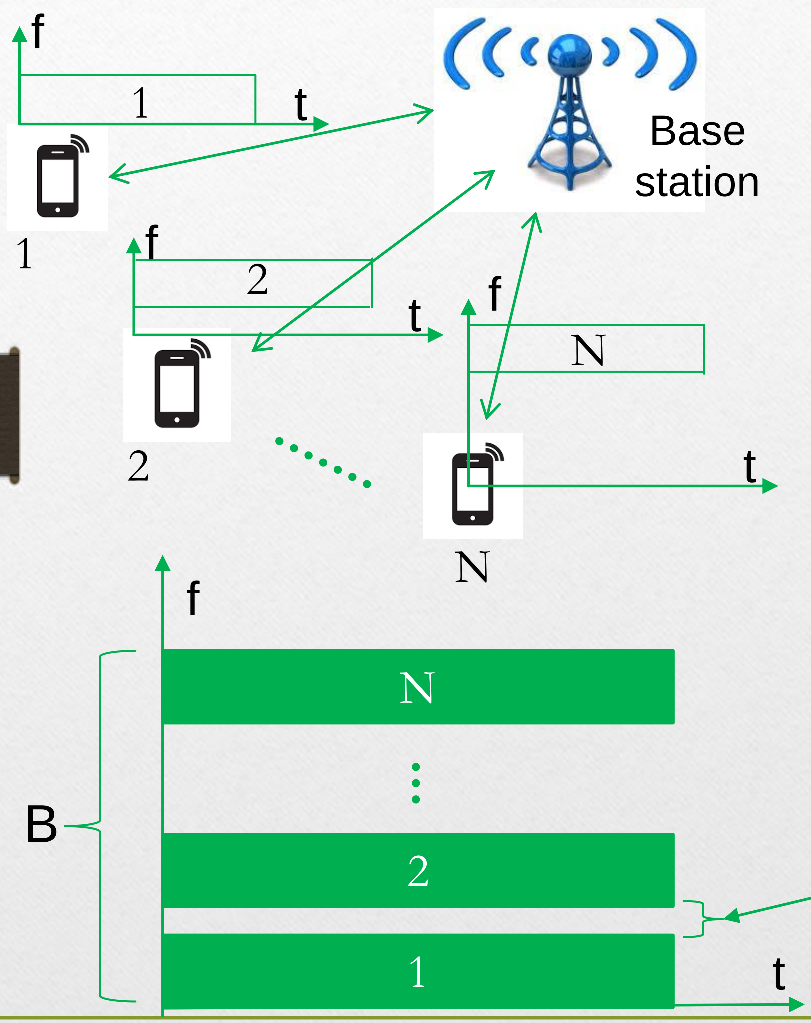
1.1. Multiplexing Techniques

- Multiplexing is a method of sharing bandwidth with other independent data channels
- There are three typical types
  - Frequency division multiplexing (FDM)
  - Time division multiplexing (TDM)
  - Code division multiplexing (CDM)



FDM TDM CDM

1.2. Frequency Division Multiplexing (FDM)



- Frequency-division multiplexing is a technique which divide total bandwidth available into series of non-overlapping frequency bands

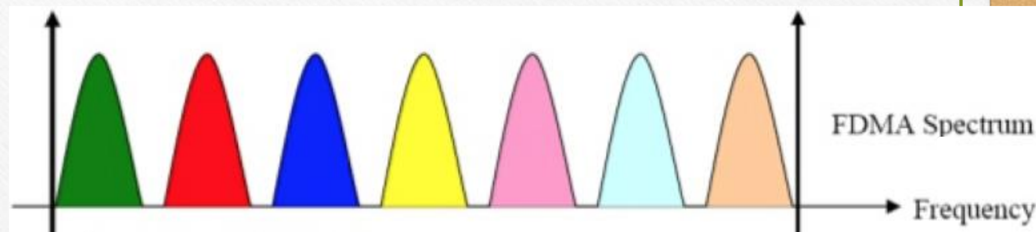
- The working frequency is divided into sub carriers
- Each sub-carrier is allocated to a channel

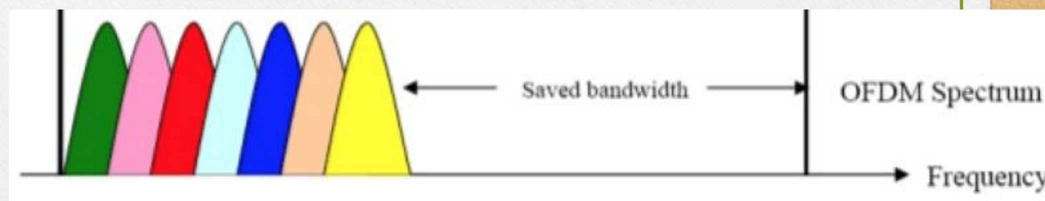
- Among the channels, there need guard bands to avoid interference each other

1.3. What is the OFDM?

How to save bandwidth

Subcarriers are overlapping to each other





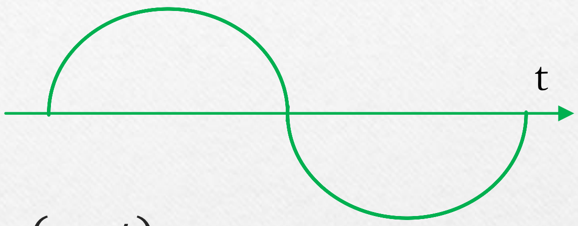
How to realize the signal at the receiver

Orthogonality

OFDM is a special case of FDM but subcarriers are orthogonal to each other

1.4. Orthogonality

- The carriers are all sine/cosine wave



- Consider the signal:

$$f(t) = sin(m\omega t)sin(n\omega t)$$

where m and n are integers, ω is angular frequency

- The integral of $f(t)$ over one period is given by

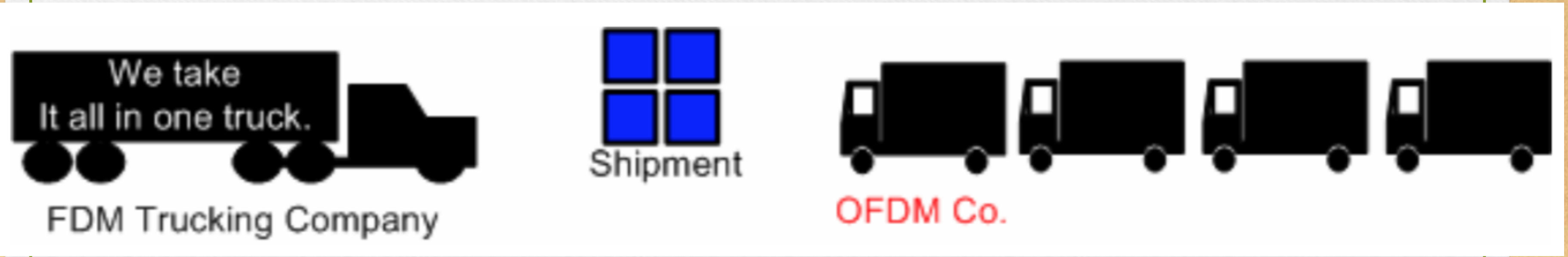
$$\int_0^{2\pi} sin(m\omega t)sin(n\omega t)dt$$

$$= \frac{1}{2}\int_0^{2\pi} \cos(m-n)\,\omega t dt - \frac{1}{2}\int_0^{2\pi} \cos(m+n)\omega t dt = 0$$

→ For all integers m and n, the carriers are orthogonal to each other

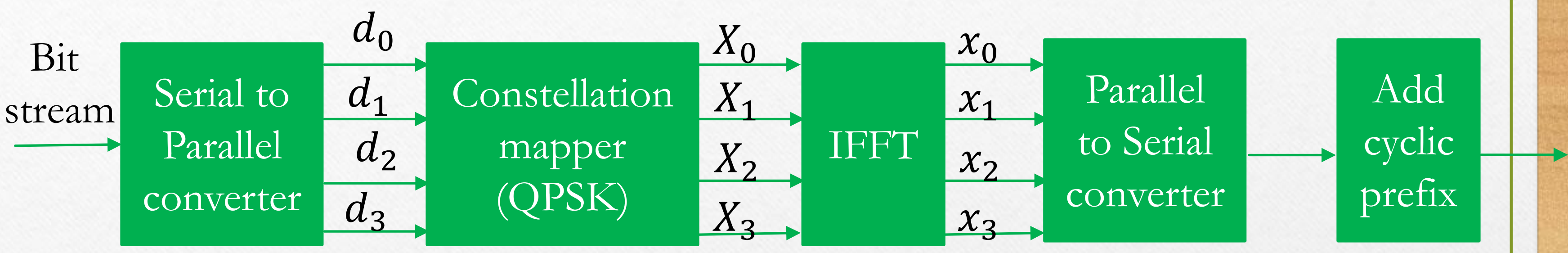
1.5. Benefits of OFDM

- Bandwidth saving
- Preventing adversely effect of frequency selective fading
- Combating intersymbol interference (ISI)



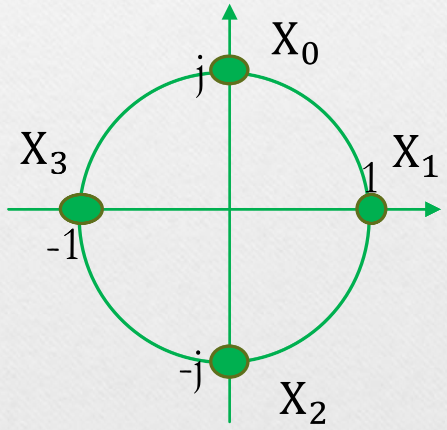
2. How does OFDM work?

2.1. OFDM Transmitter



Bit stream:
10000111

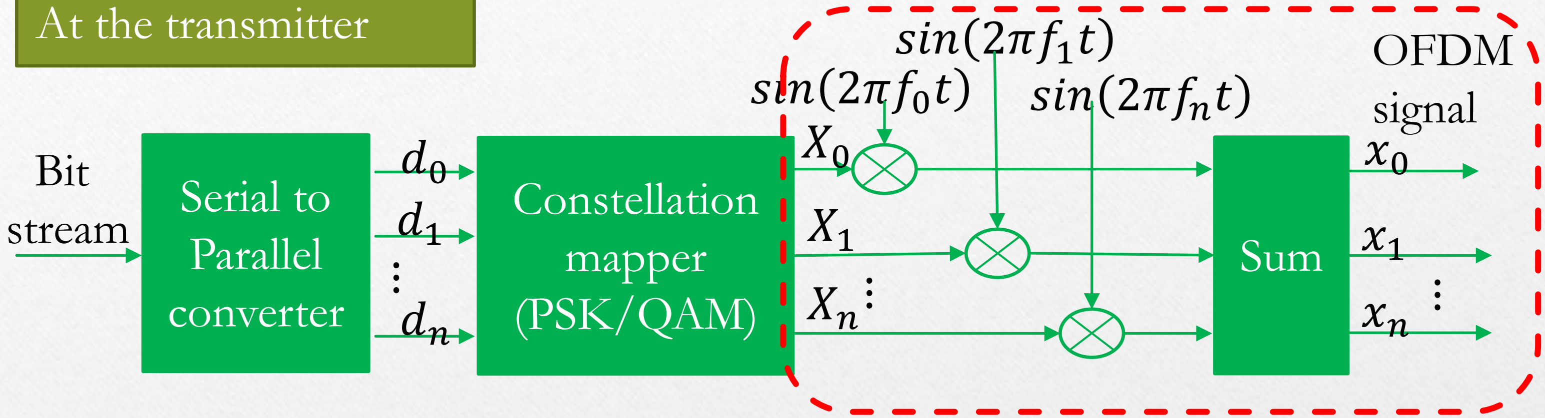
$d_0 = 11$
$d_1 = 10$
$d_2 = 00$
$d_3 = 01$

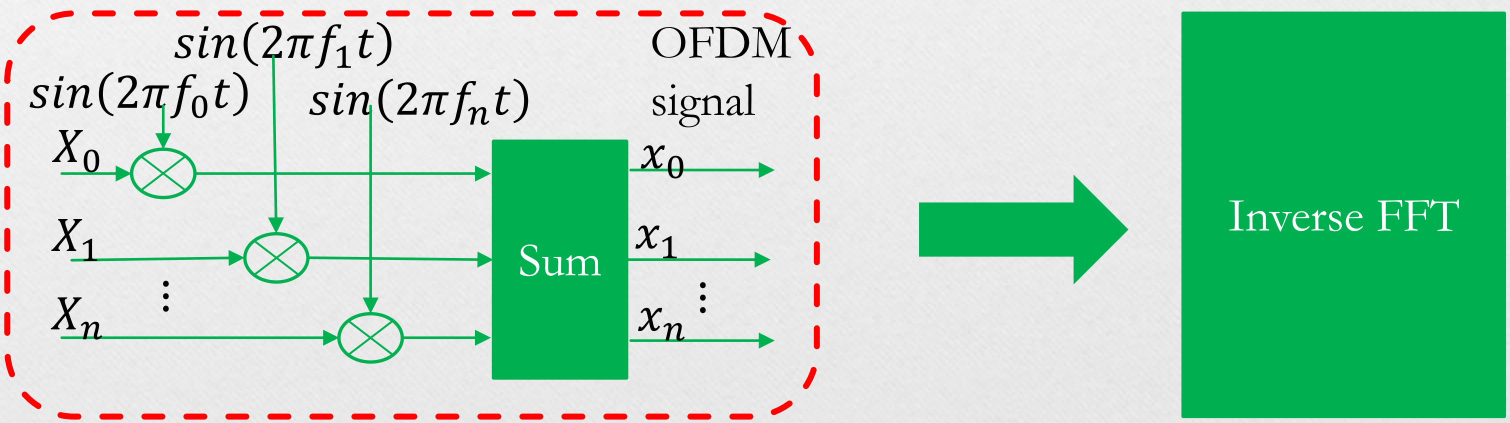


$X_0 = j$
$X_1 = 1$
$X_2 = -j$
$X_3 = -1$

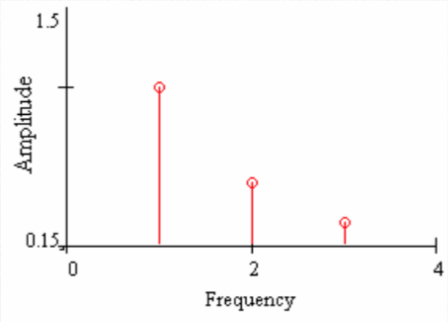
2.2. Implementing IFFT algorithm

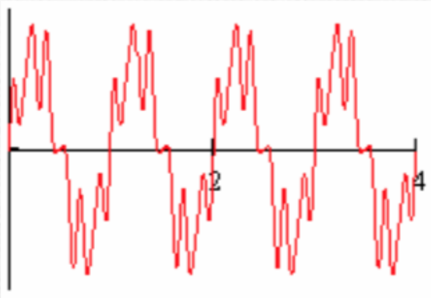
At the transmitter





2.2. Implementing IFFT algorithm





$$x_n = \sum_{n=0}^{N-1} X_k e^{j\frac{2\pi kn}{N}}$$

x_n is the value of signal at time n
X_k is the transmitted symbol
k is the k-th subcarrier
N is the number of subcarrier

$$X_0 = j \qquad x_0 = 0$$
$$X_1 = 1 \qquad x_1 = 4\mathrm{j}$$
$$X_2 = -j \qquad x_2 = 0$$
$$X_3 = -1 \qquad x_3 = -1 + \mathrm{j}$$

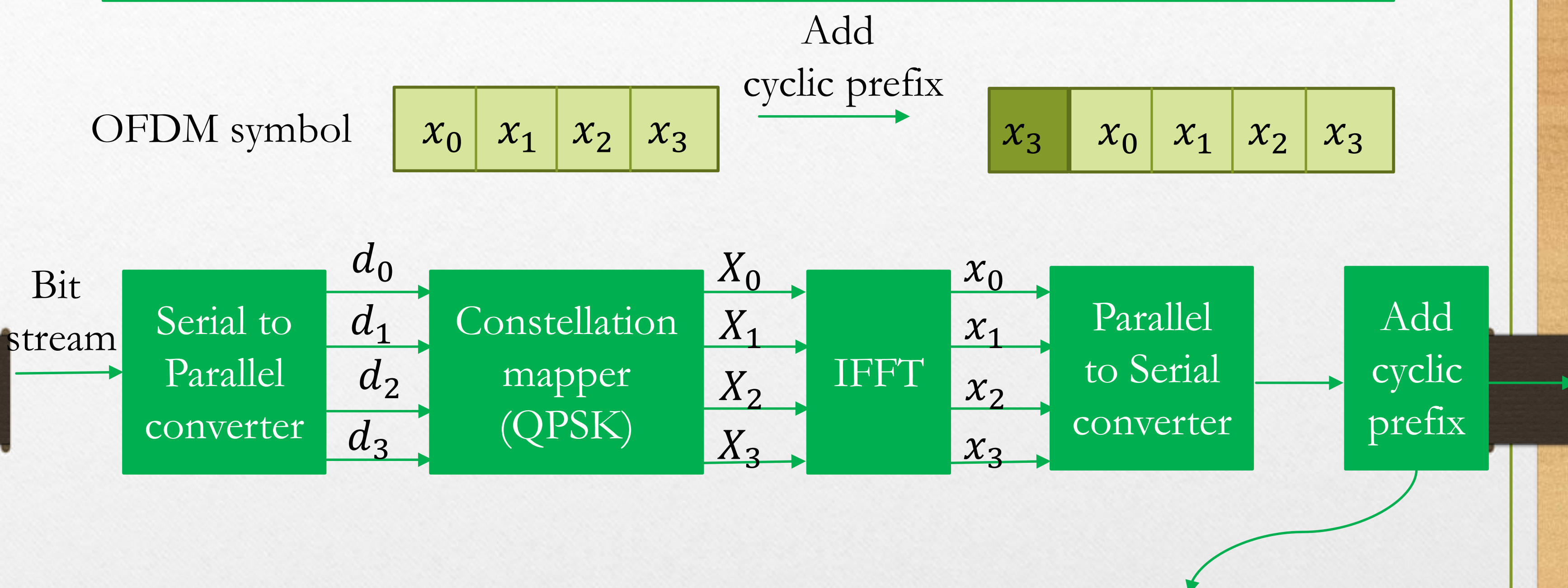
$$x_1 = \sum_{n=0}^{3} \mathrm{X_k\, e}^{\mathrm{j}\frac{2\pi \mathrm{k}1}{4}} = \sum_{n=0}^{3} \mathrm{X_k\, e}^{\mathrm{j}\frac{2\pi \mathrm{k}}{4}}$$

$$= \left[\mathrm{X_0 e}^{\mathrm{j}\frac{2\pi 0}{4}} + \mathrm{X_1 e}^{\mathrm{j}\frac{2\pi 1}{4}} + \mathrm{X_2 e}^{\mathrm{j}\frac{2\pi 2}{4}} + \mathrm{X_3 e}^{\mathrm{j}\frac{2\pi 3}{4}}\right]$$

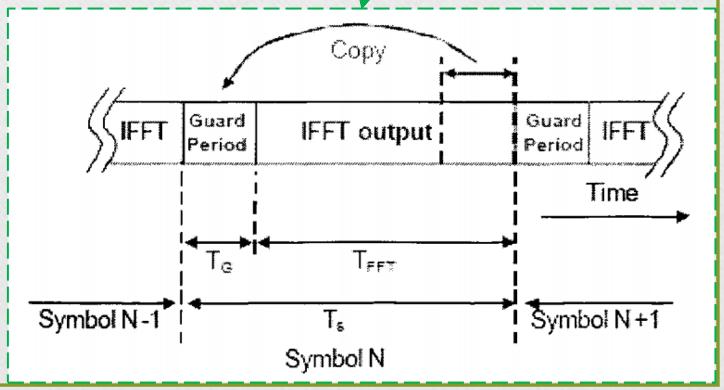
$$= [\mathrm{X_0} + \mathrm{j\, X_1} + (-1)\mathrm{X_2} + (-\mathrm{j})\mathrm{X_3}]$$

$$= [\mathrm{j} + \mathrm{j}\ 1 + (-1)(-\mathrm{j}) + (-\mathrm{j})(-1)] = 4\mathrm{j}$$

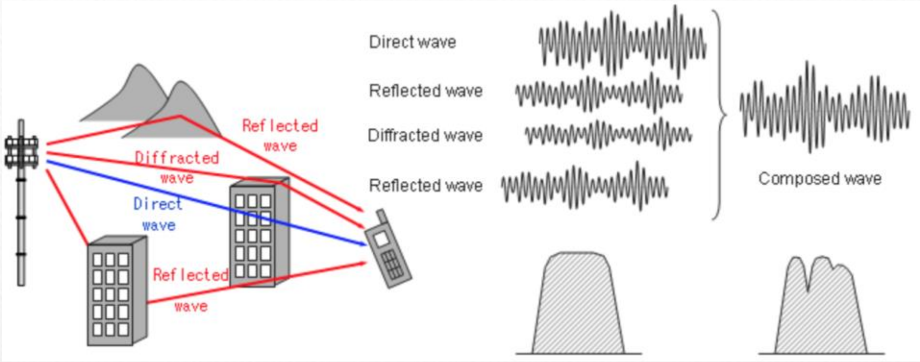
2.3. Cyclic Prefix (CP)



- Cyclic prefix (CP) is a copy of the end of the OFDM symbol
- Combating ISI and completely eliminate need for equalizers are two important advantages of the CP



2.4. Why do We Need Cyclic Prefix

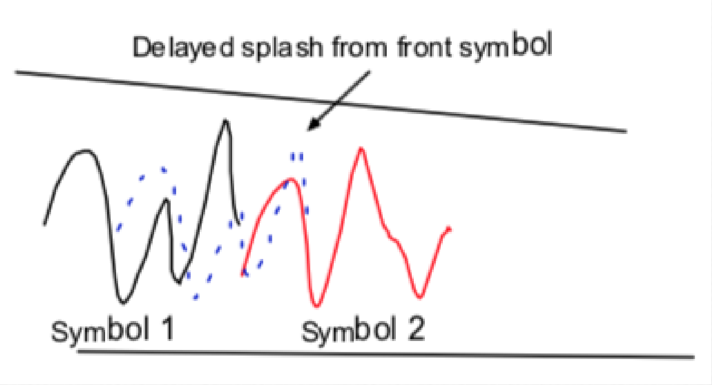


$T_{delay\_spread} > T_{symbol}$

→ Selective frequency fading

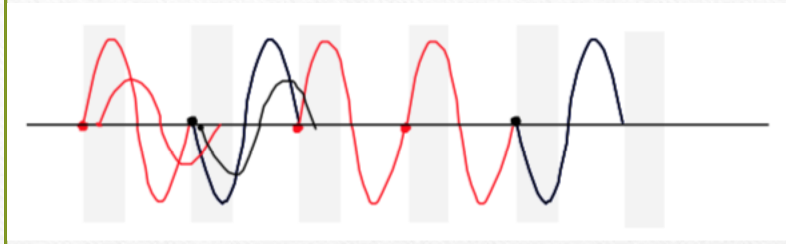
$T_{delay\_spread} < T_{symbol}$

→ Flat fading

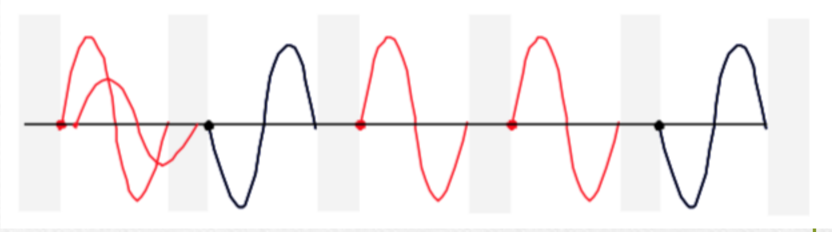


Insert something between symbols

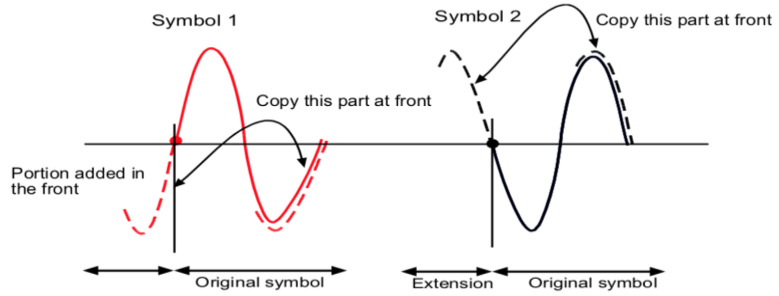
2.5. How does Cyclic Prefix Work?



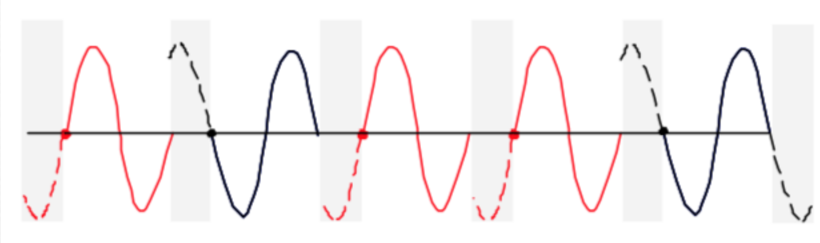
(A) The signal and its delayed version. Gray region represents for delay spread



(B) A little bit space is created by moving the symbol back. But, the blanks cannot exist in the signal



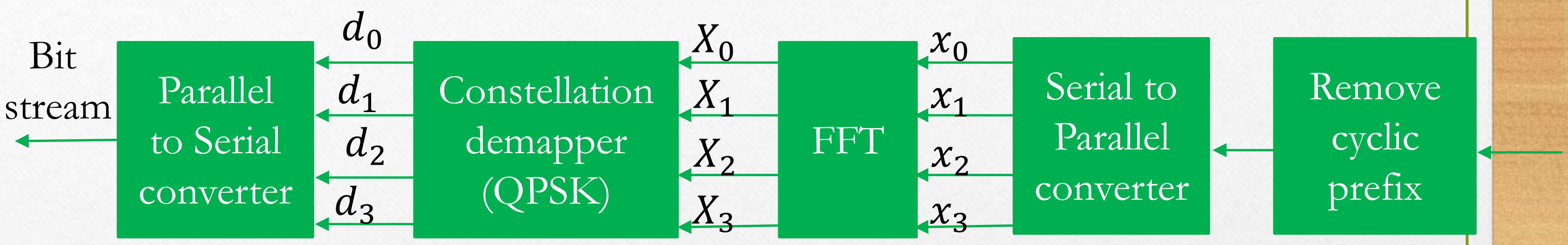
(C) The tail of the symbols is copied and pasted in front of the symbols



$$T_{OFDM\_symbol} = T_{IFFT} + T_{CP}$$

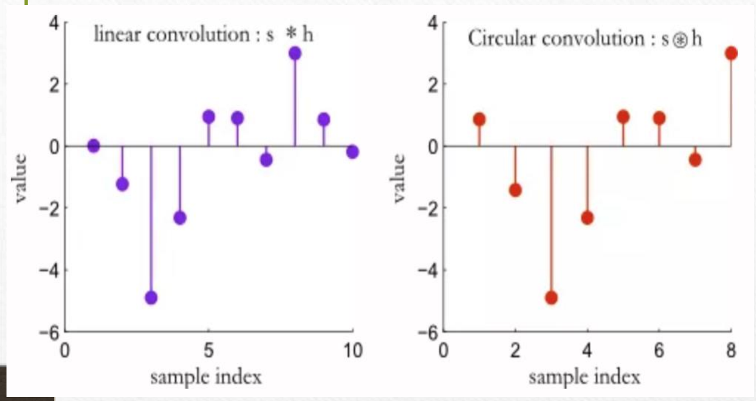
(D) The continuous signal avoids ISI

2.6. OFDM Receiver



- The cyclic prefix is removed
- The FFT of each symbol is then taken to find the original transmitted spectrum
- The data words are then combined back to the same word size as the original data

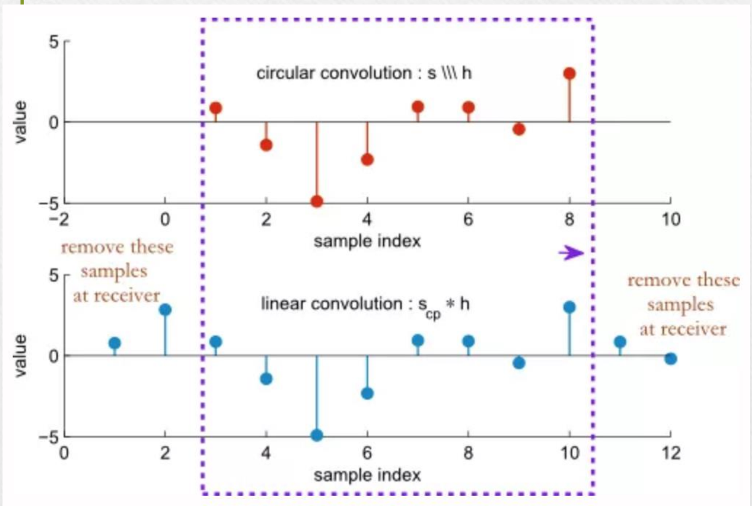
2.7. How to remove cyclic prefix



Linear convolution
$r[n] = s[n] * h[n]$

Circular convolution
$r[n] = s_{cp}[n] * h[n]$

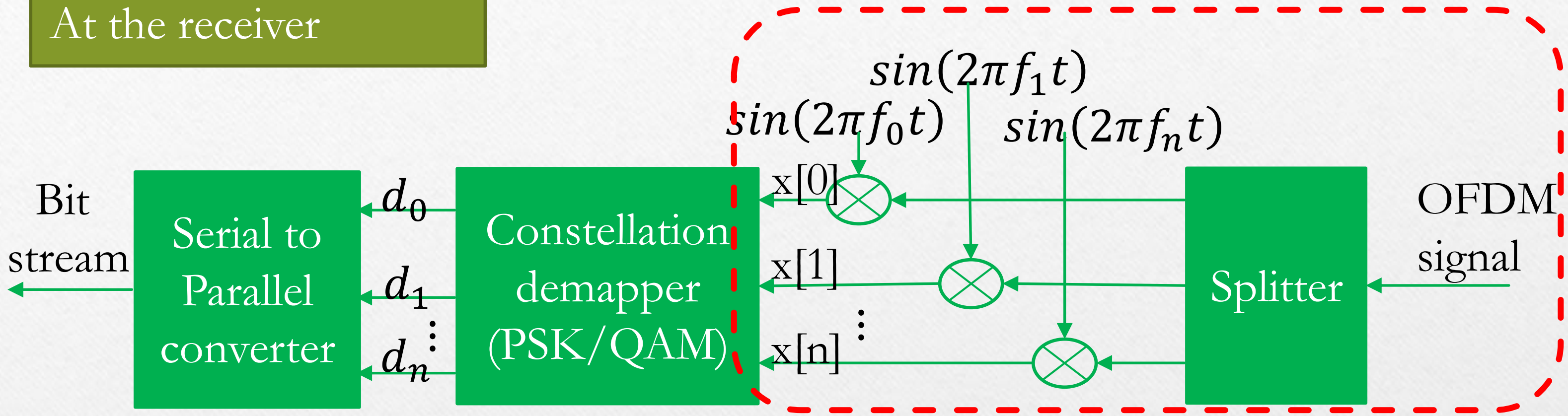
r[n] is the received signal
s[n] is the signal without CP
$s_{cp}[n]$ is the signal with CP
h[n] is the channel impulse response

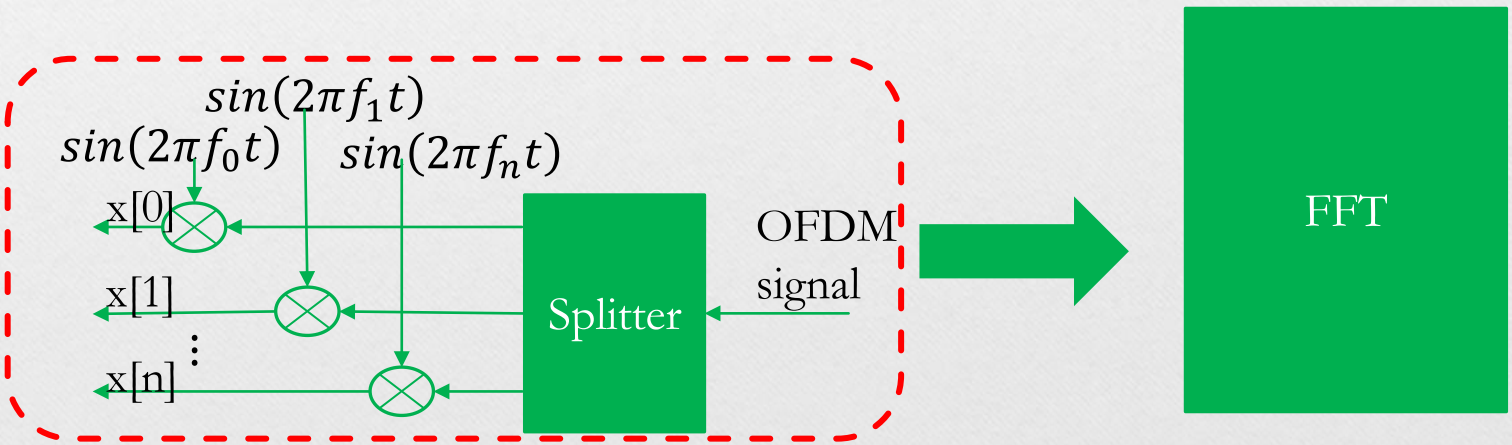


At the receiver, to remove CP part out of OFDM symbol, we just compare output between linear convolution and circular convolution. We will keep similar part and remove different parts.

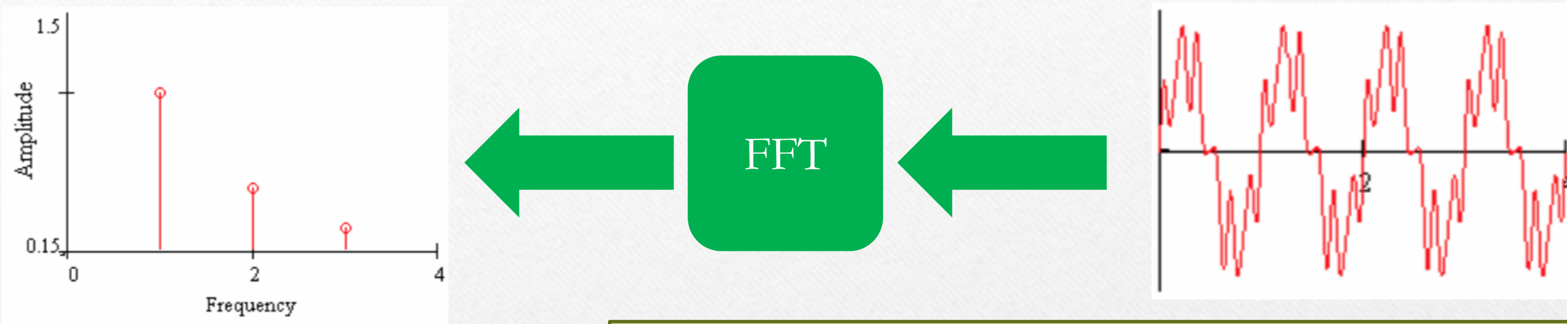
2.8. Implementing FFT alrogrithm

At the receiver





2.9. Fast Fourier Transform (FFT)



$$X_k = \sum_{n=0}^{N-1} x_n e^{-j\frac{2\pi kn}{N}}$$

x_n is the value of signal at time n
X_k is the value of spectrum in the frequency domain
k, n is the index of frequency and time
N is the number of subcarrier

$$X_0 = j \qquad x_0 = 0$$
$$X_1 = 1 \qquad x_1 = 4\mathrm{j}$$
$$X_2 = -j \qquad x_2 = 0$$
$$X_3 = -1 \qquad x_3 = -1 + \mathrm{j}$$

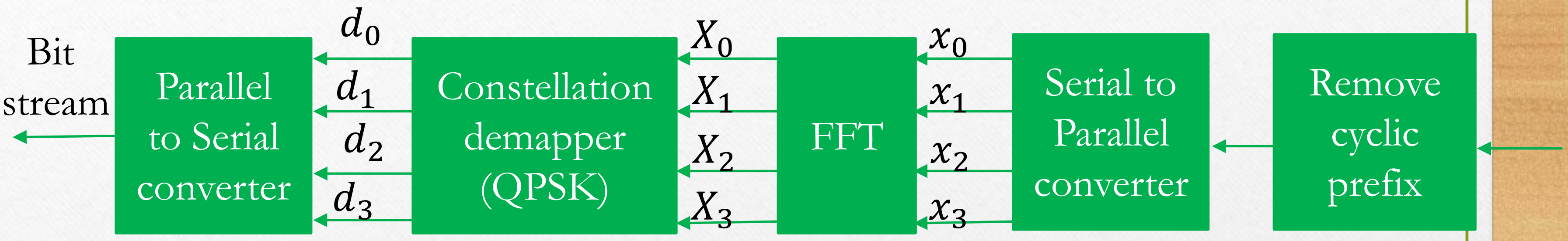
$$\mathrm{X}_1 = \sum_{\mathrm{n}=0}^{3} \mathrm{x_n}\, \mathrm{e}^{-\mathrm{j}\frac{2\pi n1}{4}} = \sum_{\mathrm{n}=0}^{3} \mathrm{x_n}\, \mathrm{e}^{-\mathrm{j}\frac{2\pi n}{4}}$$

$$= \left[x_0 \mathrm{e}^{-\mathrm{j}\frac{2\pi 0}{4}} + x_1 \mathrm{e}^{-\mathrm{j}\frac{2\pi 1}{4}} + x_2 \mathrm{e}^{-\mathrm{j}\frac{2\pi 2}{4}} + x_3 \mathrm{e}^{-\mathrm{j}\frac{2\pi 3}{4}}\right]$$

$$= [x_0 + (-\mathrm{j})\, x_1 + (-1)x_2 + jx_3]$$

$$= [0 + (-j)4j + 0 + j(-1 + j)] = 3 - j$$

2.10. Get Back Original Data



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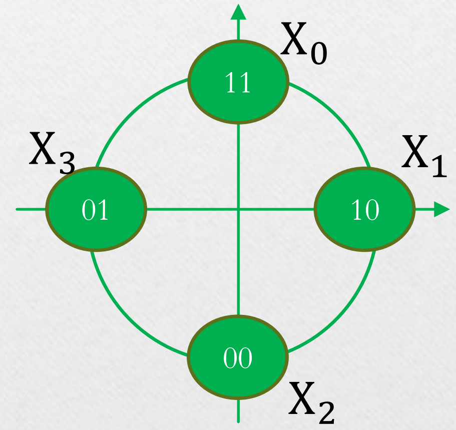
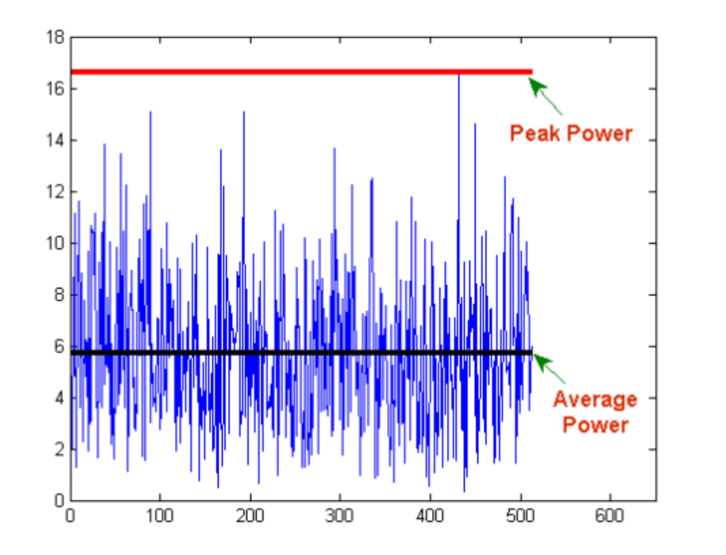


$X_0 = j$
$X_1 = 1$
$X_2 = -j$
$X_3 = -1$

3. Difficulties of OFDM

3.1. Peak-to-Average Power Ratio (PAPR)

PAPR is the ratio of peak power to the average power of a signal

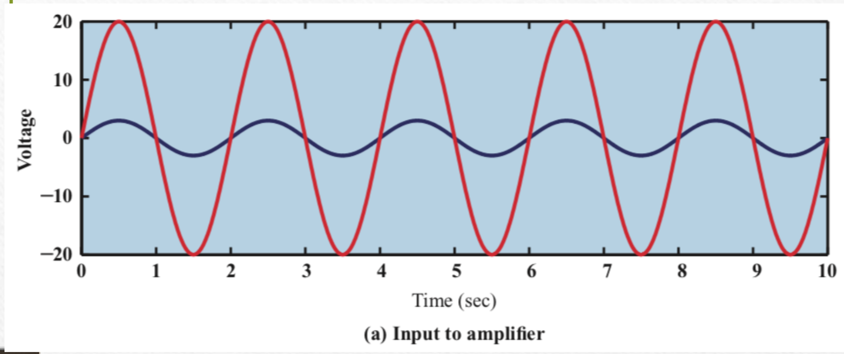


PAPR occurs when in a multicarrier system the different sub-carriers are out of phase with each other.

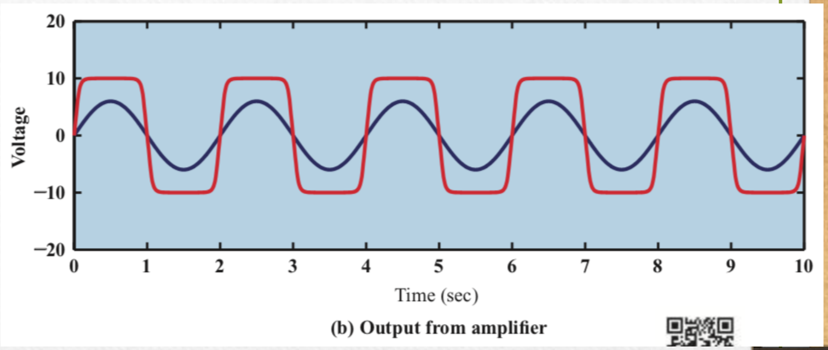
The peak value of the signal is substantially larger than the average value

PAPR becomes one of the most important challenges for implementation of OFDM

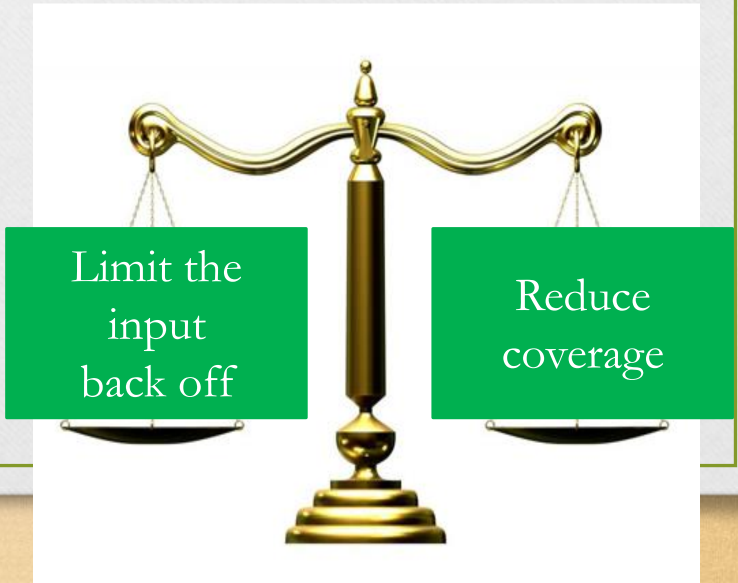
3.1. Peak-to-Average Power Ratio (PAPR)



(a) Input to amplifier



(b) Output from amplifier



3.2. Intercarrier Interference (ICI)

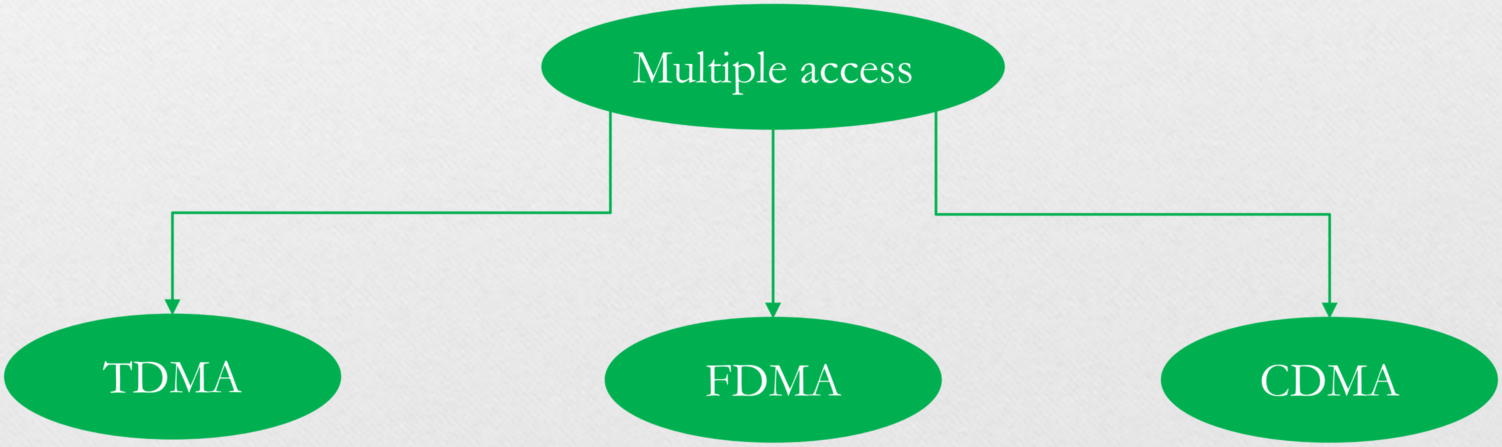
- ICI is caused by two reasons:
  - Delay spread of radio channel exceeds the CP interval
  - The frequency offset at the receiver (Doppler shift)

4. Orthogonal Frequency Division Multiple Access (OFDMA)

4.1. What is the Multiple Access?

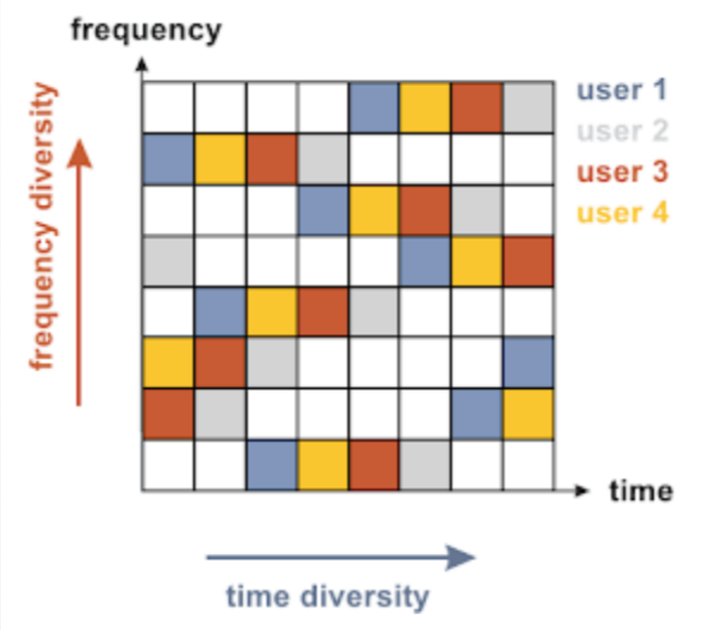
A spectrum is limited → The sharing is necessary → To improve the capacity

Multiple access is carried out by permitting the available bandwidth to be used simultaneously by different users

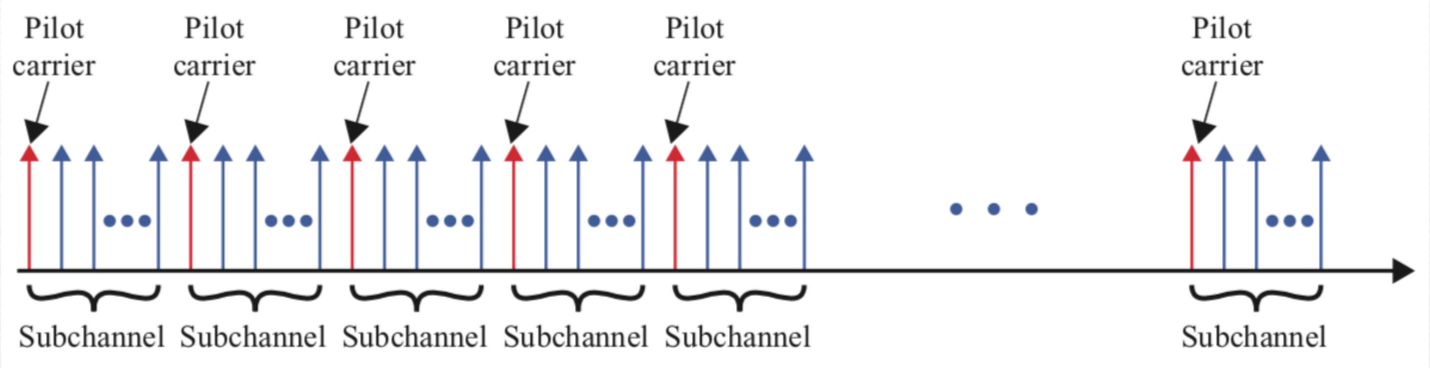


4.2. What is the OFDMA?

- **FDMA** is a technique that frequency band split into small frequency channels and different channels are assigned to different users
- **TDMA** is a technique that every user is permitted to transmit at the same frequency band at different times
- **OFDMA** is a combination of FDMA and TDMA, then, it allows multiple access on the same channel by using OFDM



4.3. Benefit of the OFDMA

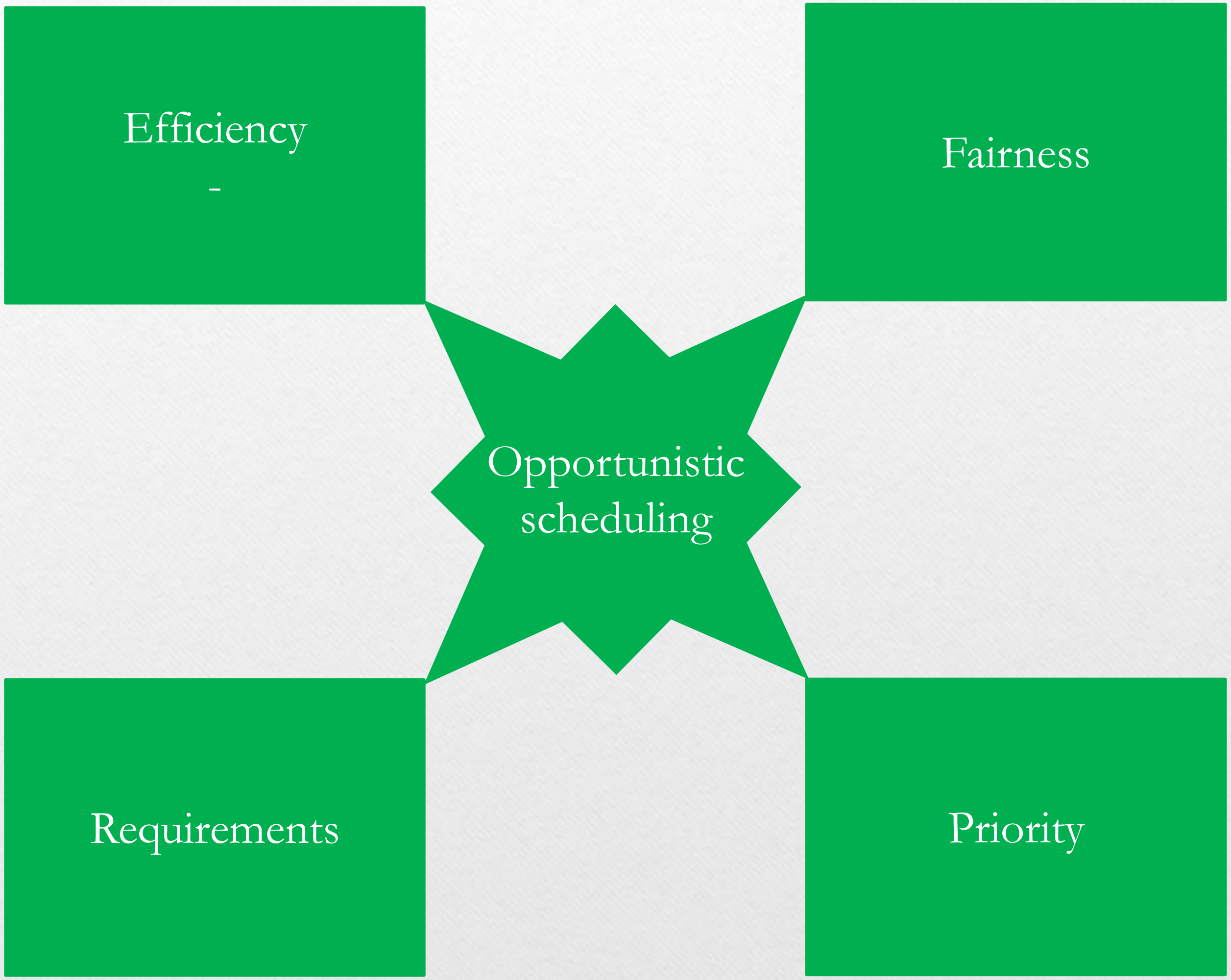


Adjacent subcarriers help to optimize a balance of channel efficiency

Regularly spaced subcarriers helps to choose good frequencies

Randomly spaced subcarriers helps to reduce adjacent-cell interference

4.4. Opportunistic Scheduling



Thank you for your attention!