Throughput Analysis of Wireless Relay Slotted ALOHA with Network Coding (NC)

Bui Hoang Anh Tuan
CCLab, The University of Aizu
16/11/2016

Reference papers

[1]. D. Umehara et. al., “Throughput Analysis of Wireless Relay Slotted ALOHA Systems with Network Coding”, proc. of IEEE GLOBECOM, 2008.

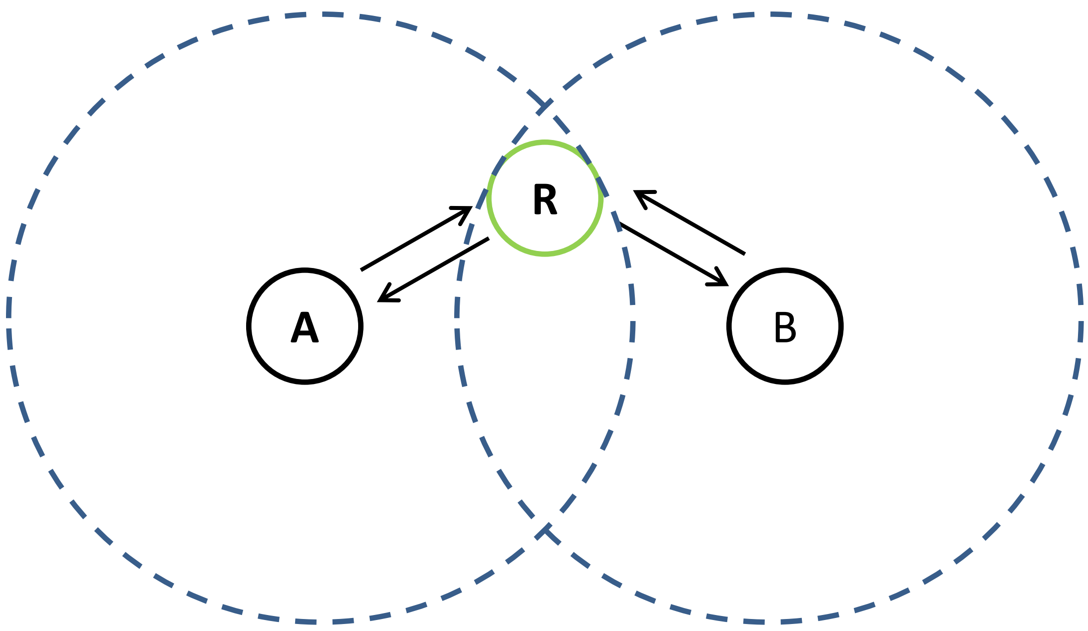
[2]. D. Umehara et. al., “Wireless Network Coding in Slotted ALOHA with Two-hop Unbalanced Traffic”, IEEE Journal on Selected Areas in Communication, 2009.

Outlines

1. System model
2. Throughput analysis: ALOHA
3. Throughput analysis: ALOHA with NC
4. Simulation results
5. Conclusion

System model

- Two end nodes (**A** and **B**), one relay node (**R**)
  - **A** & **B** are out of each other's range, but in range of **R** → communicate via **R** (**R** has buffer to store packets).



System model (cont.)

- Assumptions (every nodes)
  - Has one omni-directional antenna, cannot transmit and receive simultaneously
  - Has unlimited buffer if needed
  - Medium access : use slotted ALOHA
    - Receiver broadcasts ACK (very short length) for each correctly received packet
  - Packet loss only caused by collision

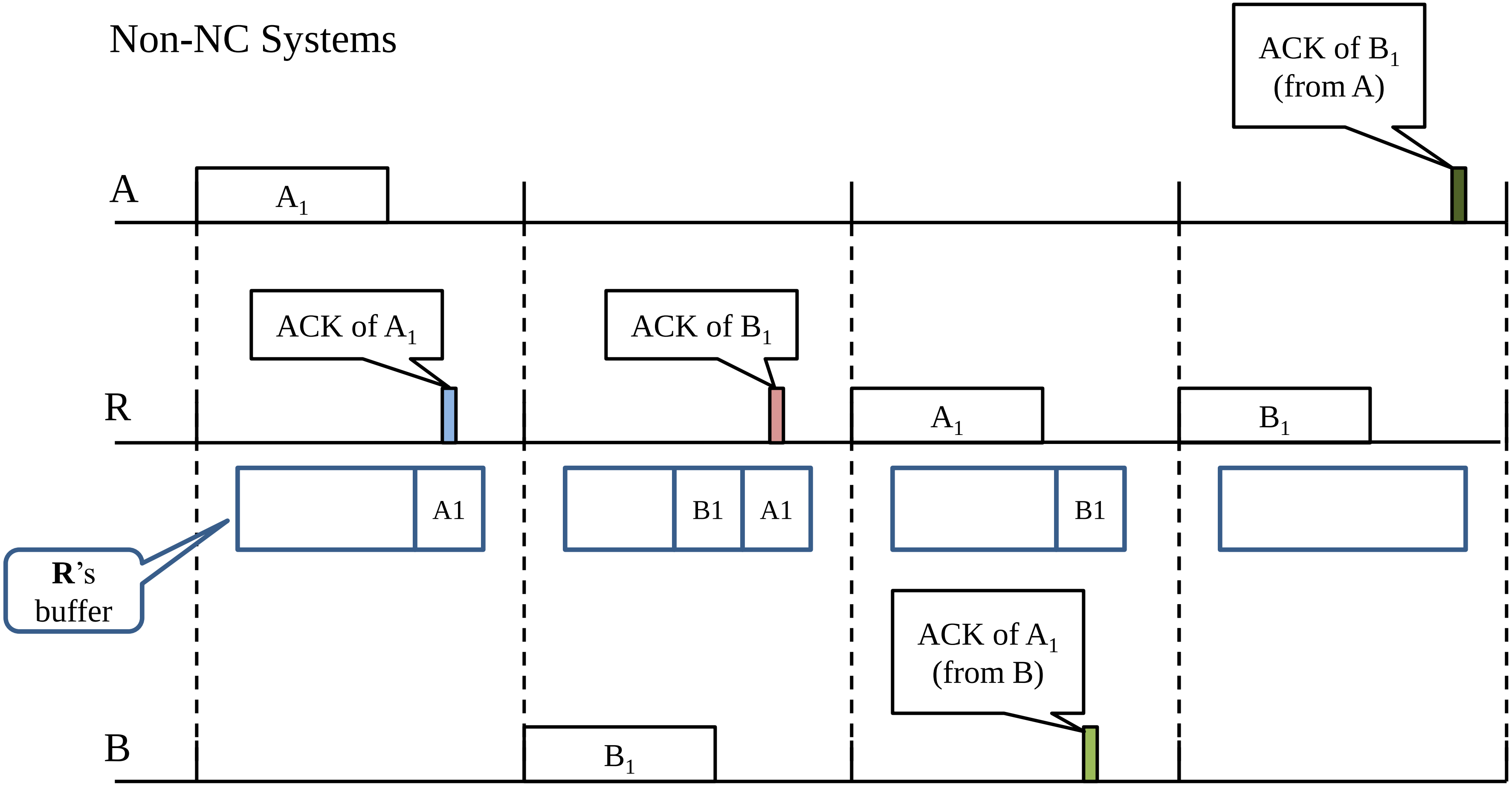
System model (cont.)

- ALOHA (end nodes):
  - In re-transmission (backlogged) mode, transmit with prob. = g_r
  - In normal mode, transmit with prob. = 1
  - ➢ Average transmissions (incl. new + re-transmission) prob. in a slot = g

System model (cont.)

- ALOHA (node **R**):
  - **R** does not generates new packet → only work in re-transmission mode
    - If R has packets in buffer, send packet in slot with probability q
- ***<u>Non-NC case</u>***:
  - If the buffer is non-empty, **R** sends the packet at its physical buffer's head

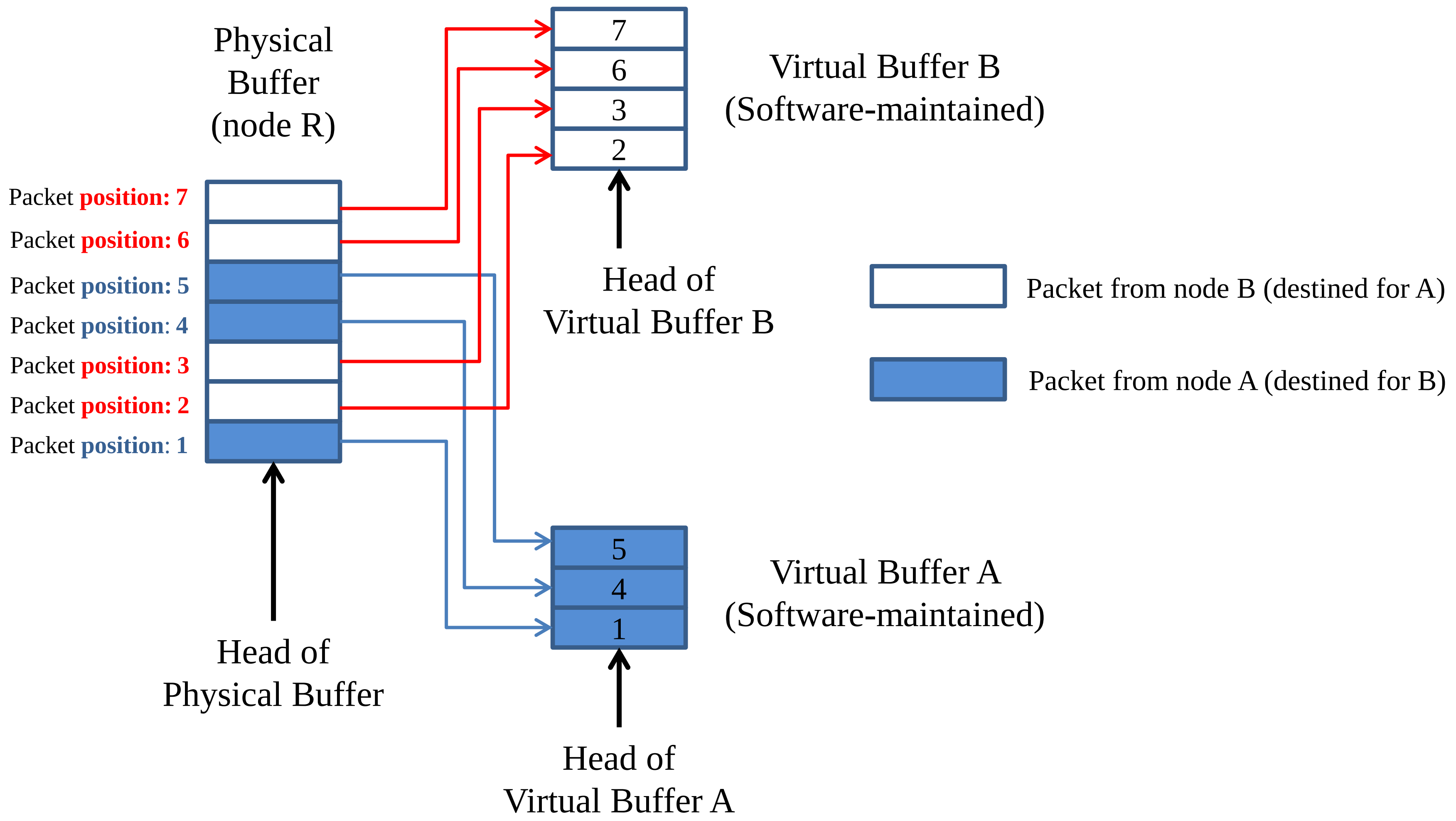
System model (cont.)



System model (cont.)

- ***NC case:***
  - **R** maintains two *virtual buffers* (**vA**, **vB**): store pointers point to positions of packets originated from **A** (and **B**, respectively) in the physical buffer

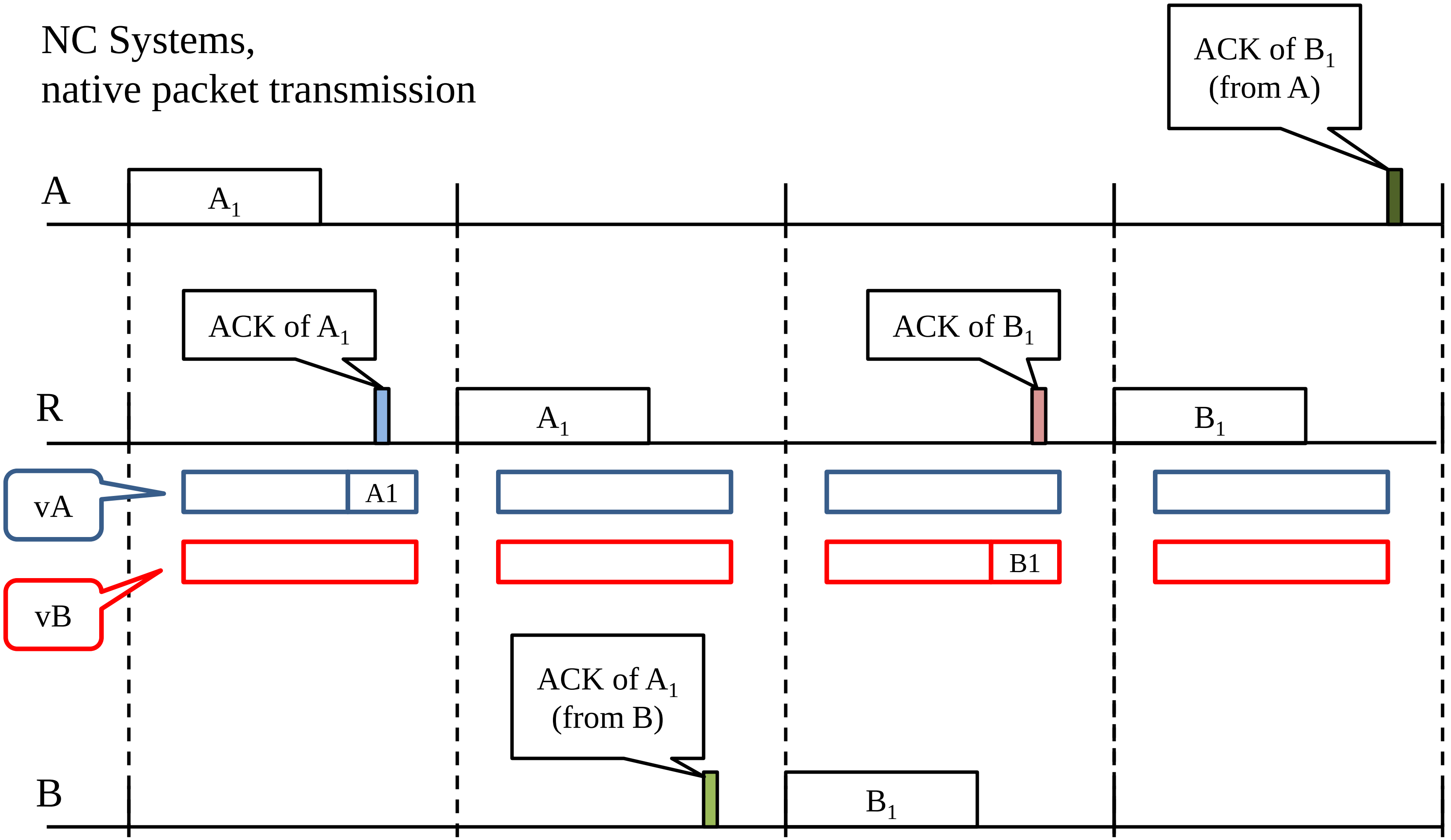
System model (cont.)



System model (cont.)

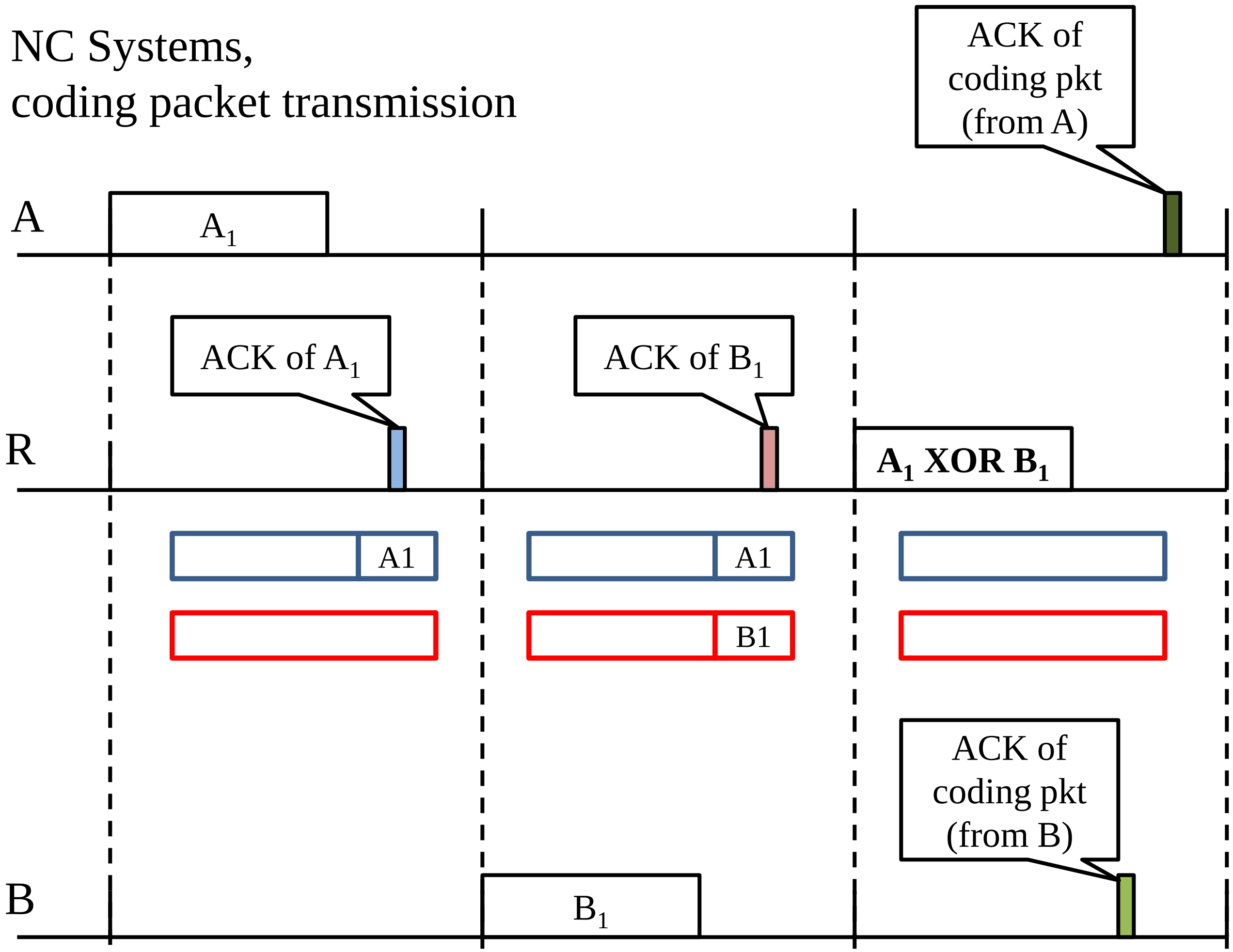
- ***NC case (cont.):***
  - If two virtual buffers are non-empty: **R** XOR packets at the head of **vA** and **vB** → create "coding packet" and sends it.
  - If only one virtual buffer is non-empty: **R** sends the packet at head of that virtual buffer (called "native packet")

System model (cont.)



System model (cont.)

NC Systems,
coding packet transmission



How does A and B decodes the coding packet "A_1 XOR B_1"?

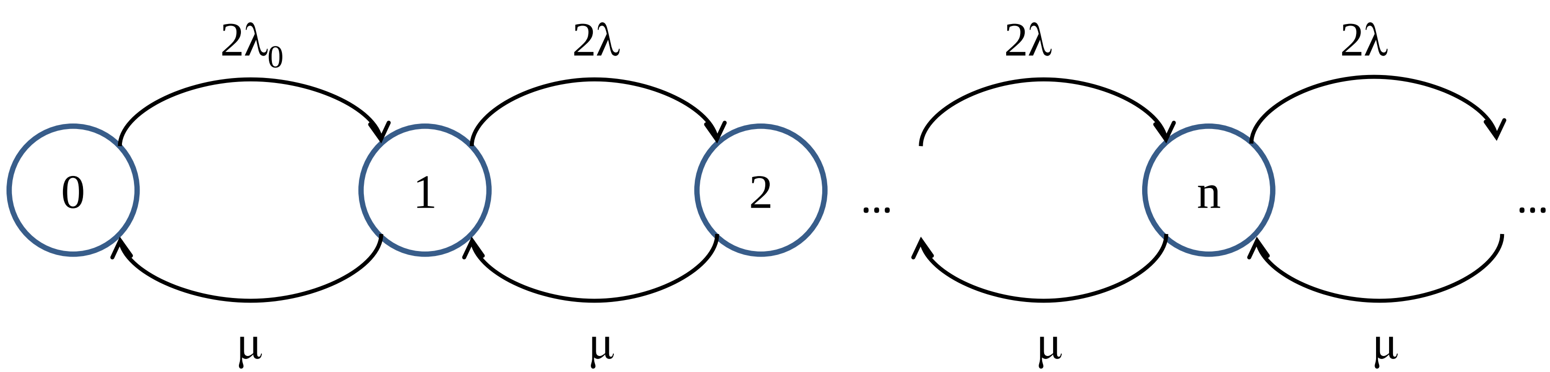
- **A** needs to get B_1: use A_1 XOR "A_1 XOR B_1" = get B_1
- **B** needs to get A_1: use B_1 XOR "A_1 XOR B_1" = get A_1

Throughput Analysis: ALOHA

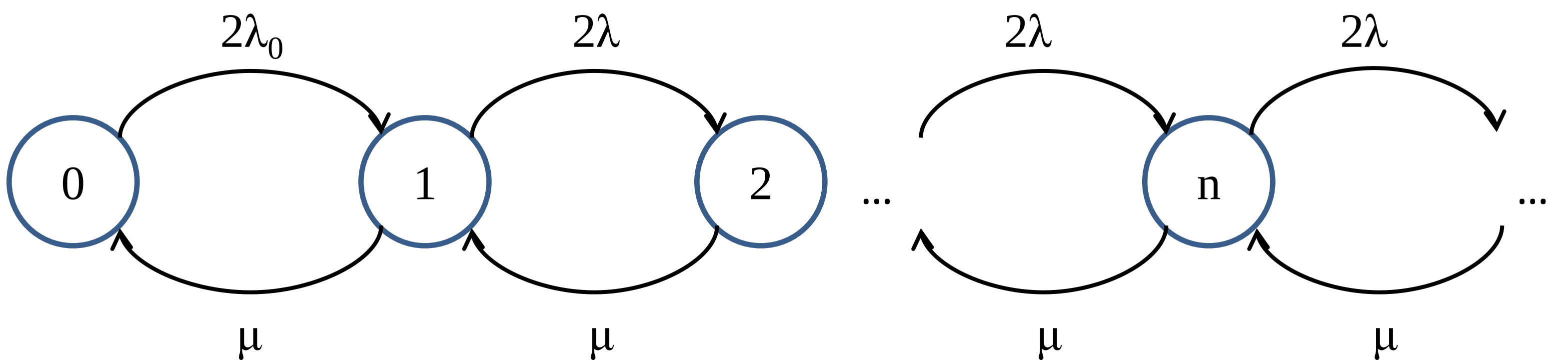
- For throughput S, we need probability of successful transmission from node **R**
- **R**'s success probability depends on # of packets in its buffer
- # of packets in **R**'s buffer at next slot only depends on that at current slot → use Discrete-Time Markov Chain (DTMC) to model

Throughput Analysis: ALOHA (cont.)

- DTMC of # of packets in **R**'s buffer (self-transitions are omitted)
- State i : there are i packets in buffer

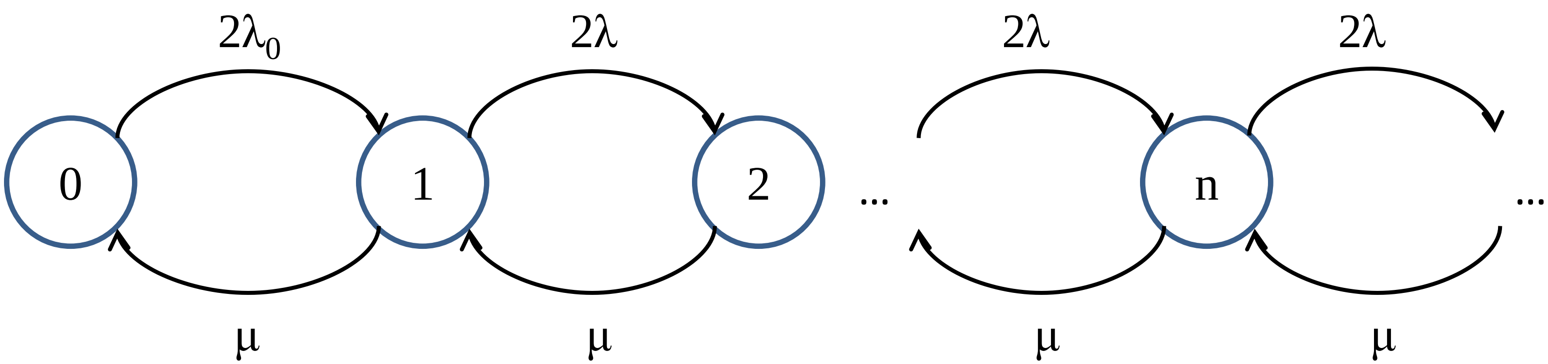


Throughput Analysis: ALOHA (cont.)



- Transition probabilities:
  - $P_{0,1}$ = only one end transmits = $2*g(1\text{-}g) = 2\lambda_0$
  - $P_{i,i+1}$ = **R** doesn't transmit & only one end transmits = $(1\text{-}q)*2\lambda_0$ = 2λ $\{i > 0\}$
  - $P_{i,i\text{-}1}$ = **R** successfully transmits a packet to an end node = **R** transmits & the packet's destination node doesn't = $q(1\text{-}g)$ = μ $\{i > 1\}$

Throughput Analysis: ALOHA (cont.)



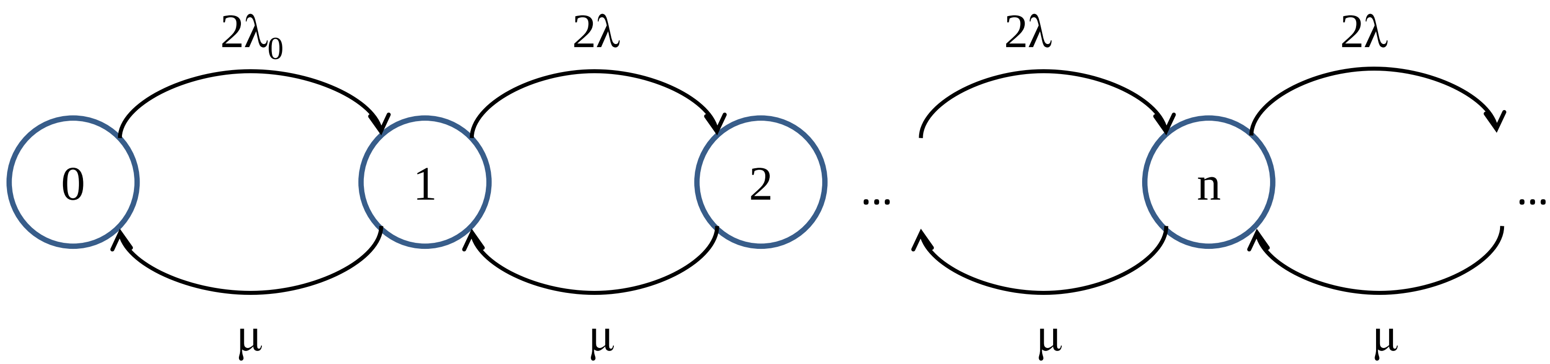
- Local balance equation (birth-death process):

$$\begin{cases} 2\lambda_0 P(0) = \mu P(1) \\ 2\lambda P(i) = \mu P(i+1) \qquad \text{where } \ i > 0 \end{cases}$$

- Normalization property: $\sum_{\forall i} P(i) = 1$

→ Find steady-state probabilities $P(i)$

Throughput Analysis: ALOHA (cont.)



- Throughput S = average number of successfully delivered packets from **R** to **A** & **B** per slot time.

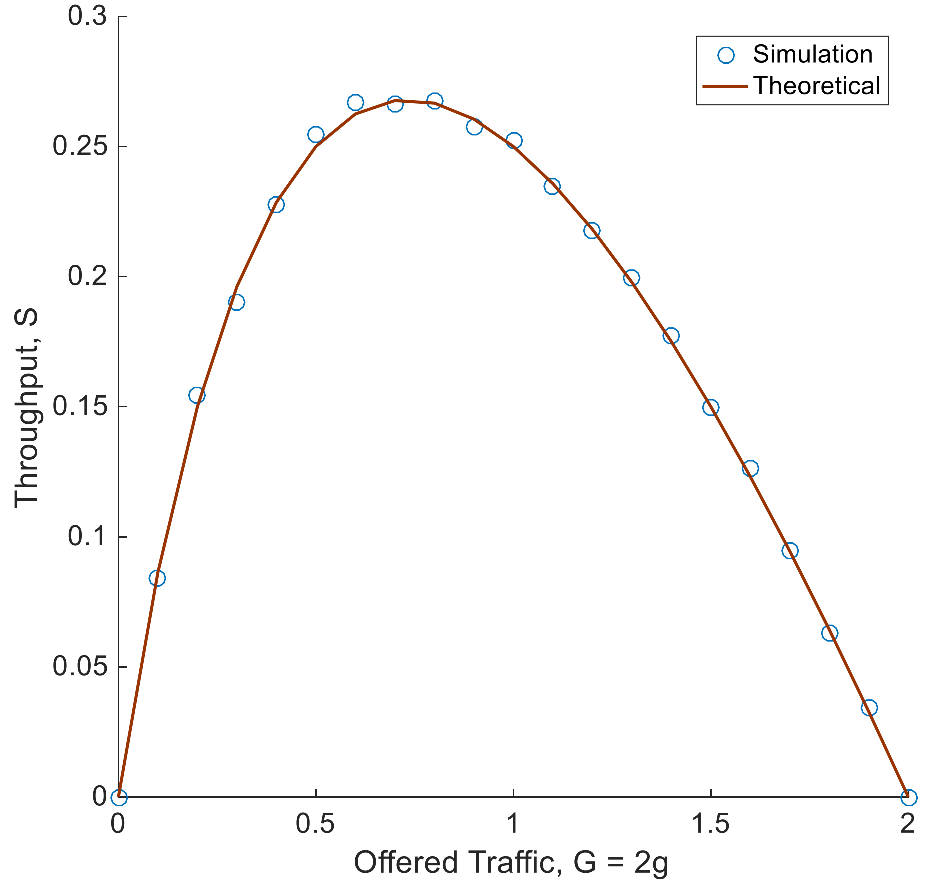
$$S = 1 * \mu\left[1 - P(0)\right] = \frac{G(1-G/2)}{1+G}, \quad \text{where } G = 2g$$

Does not depend on q

- Stabilization condition: $2\lambda < \mu$

Throughput Analysis: ALOHA (cont.)

- Simulation results of non-NC ALOHA

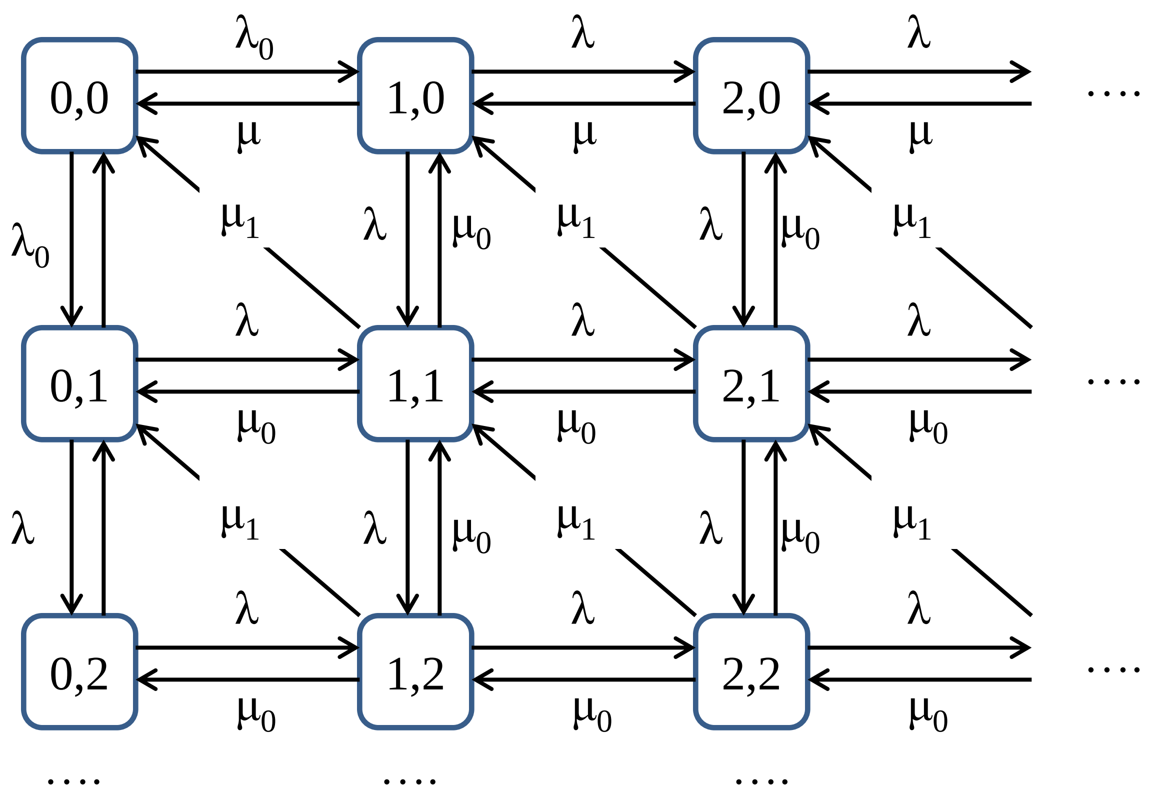


Throughput Analysis: ALOHA with NC (cont.)

- Similarly, we need to model the number of packets in **R**'s buffer using DTMC to find throughput $\boldsymbol{S}$.
- Pair of numbers of packets inside **vA** and **vB** in a slot i.e., (A_k, B_k) matters
- $(A_k, B_k) = (\neq 0, \neq 0)$ leads to coding opportunity → need two-dimension DTMC to model (A_k, B_k)

Throughput Analysis: ALOHA with NC (cont.)

- Two-dimension DTMC of (A_k, B_k) (self-transitions are omitted)

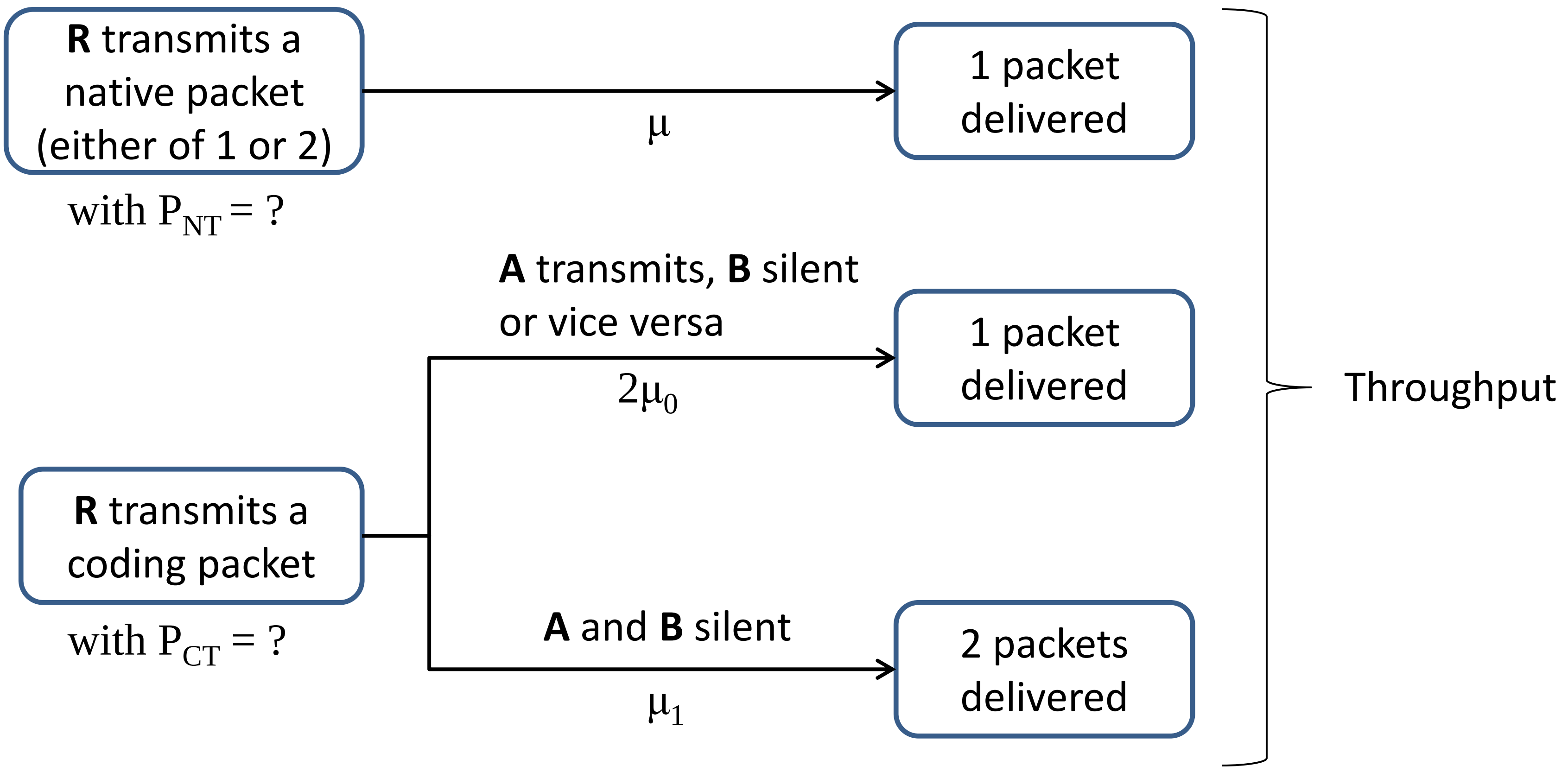


Throughput Analysis: ALOHA with NC (cont.)

- Transition probabilities: λ_0, λ, μ same as non-NC
  - μ_0 = **R** transmits a coding packet & only one silent node (i.e., only one can receive the coding packet) $= qg(1\text{-}g)$
  - μ_1 = **R** transmits a coding packet & both nodes do not transmit (i.e., both can receive the coding packet) $= q(1\text{-}g)^2$
  - Note that $\mu_0 + \mu_1 = \mu$

Throughput Analysis: ALOHA with NC (cont.)

- How to calculate throughput?



Throughput Analysis: ALOHA with NC (cont.)

$$\rightarrow S = 1 * \mu P_{NT} + \left(2\mu_0 + \mu_1 * 2\right) P_{CT} = \mu P_{NT} + 2\mu P_{CT}$$

$$\begin{cases} P_{NT} = \left(P_A\left(0\right) - P(0,0)\right) + \left(P_B\left(0\right) - P(0,0)\right) = P_A\left(0\right) + P_B\left(0\right) - 2P(0,0) \\ P_{CT} = 1 - P(0,0) - P_{NT} = 1 - P_A\left(0\right) - P_B\left(0\right) + P(0,0) \end{cases}$$

- Must find steady-state probabilities $P(n,m)$. Then,
  - $P_A(0)$ can be find by summing $P(0,m)$ over all m
  - Similar, $P_B(0)$ can be find summing $P(n,0)$ over all n
  - But finding $P_A(0)$ and $P_B(0)$ that way is less accurate. Why?

Throughput Analysis: ALOHA with NC (cont.)

- Finding $P(m,n)$: relax (approximate) local balance equation to apply on adjacent joint prob. instead of adjacent marginal prob.

$$\begin{cases} \mu P(n+1,m) = \mu P(n,m+1) = \lambda P(n,m) \quad \text{where } (n,m) \neq (0,0) \\ \\ \mu P(1,0) = \mu P(0,1) = \lambda_0 P(0,0) \end{cases}$$

- Normalization properties: $\sum_{\forall n,m} P(n,m) = 1$

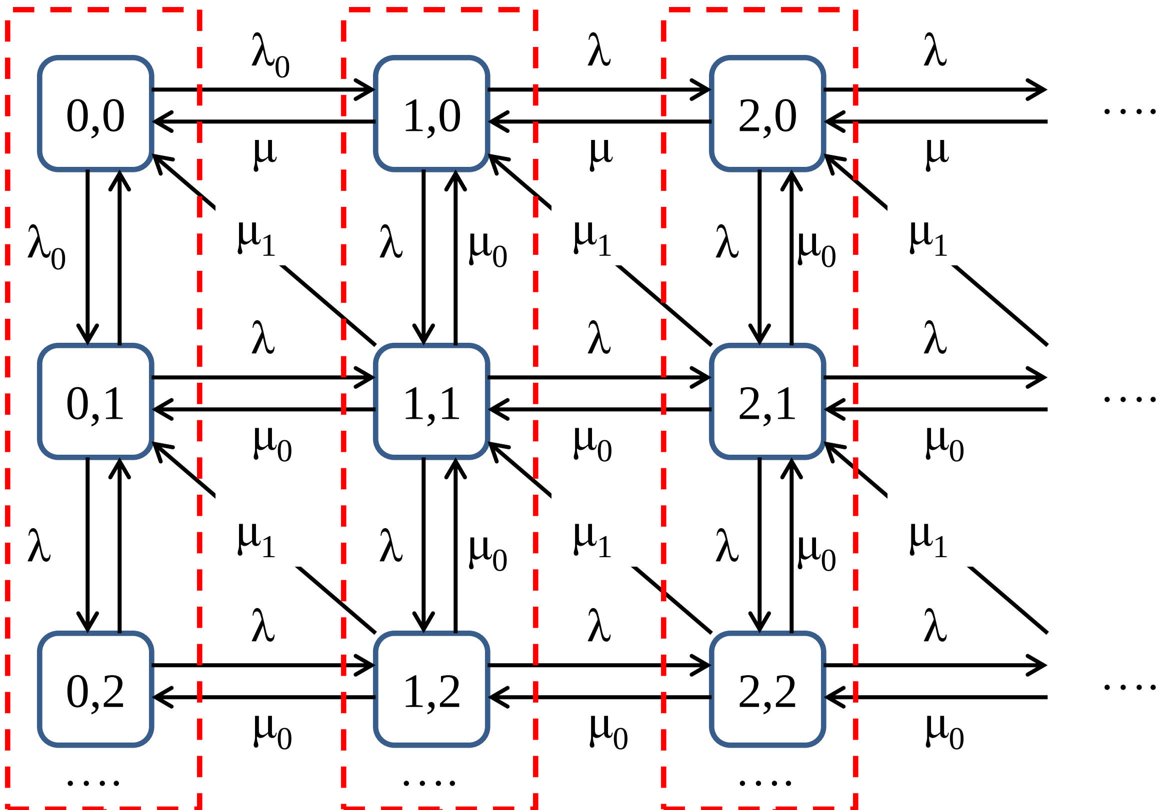
→ Find $P(n,m)$ (including $P(0,0)$)

Throughput Analysis: ALOHA with NC (cont.)

- Back to the question:
  - But finding $P_A(0)$ = sum[$P(0,m)$]over m & $P_B(0)$ = sum[$P(n,0)$] over n is less accurate. Why?
  - Because relations between all $P(n,m)$ is established using *approximation* => using that way adds up approximation errors
- The most accurate way to calculate $P_A(0)$ and $P_B(0)$?

Throughput Analysis: ALOHA with NC (cont.)

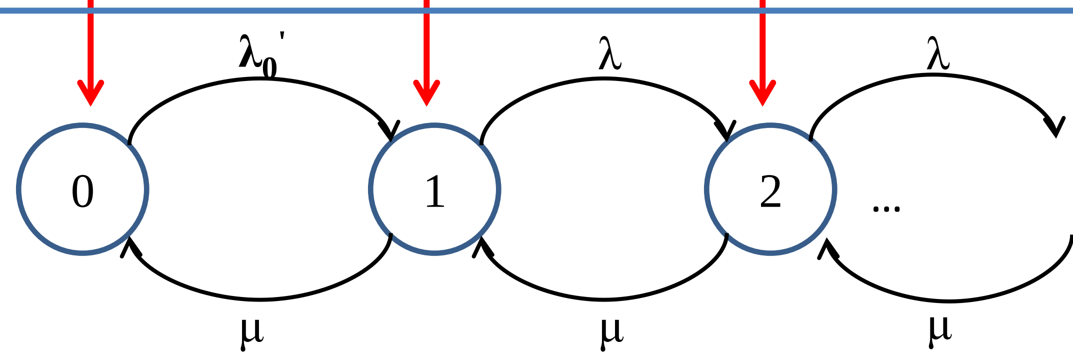
DTMC for the pair (A_k, B_k)



A_k : # of pkts in **vA** in k-th slot

B_k : # of pkts in **vB** in k-th slot

DTMC for A_k



Throughput Analysis: ALOHA with NC (cont.)

- Transition probabilities:
  - μ same as non-NC: if **R** sends a packet to **B**, prob. that the packet reaches **B** (i.e., **vA** has one less packet) only depends on whether **B** is silent, regardless of coding or native packet. Alternative argument: $\mu = \mu_0 + \mu_1$
  - $\lambda_0' = P\left(A_{k+1} = 1 \mid A_k = 0\right)$

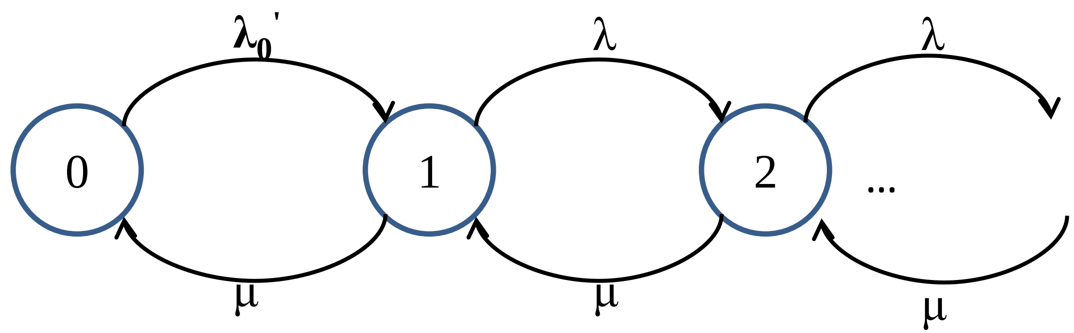
$$= P\left(A_{k+1} = 1 \mid A_k = 0 \cap B_k = 0\right) P\left(B_k = 0 \mid A_k = 0\right) +$$

$$P\left(A_{k+1} = 1 \mid A_k = 0 \cap B_k \neq 0\right) P\left(B_k \neq 0 \mid A_k = 0\right)$$

$$= \lambda_0 \frac{P(0,0)}{P_A(0)} + \lambda \left(1 - \frac{P(0,0)}{P_A(0)}\right)$$

Throughput Analysis: ALOHA with NC (cont.)

DTMC for A_k



- Local balance equation for DTMC of A_k:

$$\begin{cases} \lambda_0' P_A(0) = \mu P_A(1) \\ \lambda P_A(i) = \mu P_A(i+1) \qquad \text{where } i > 0 \end{cases}$$

Depends on $P(0,0)$

- Normalization property: $\sum_{\forall i} P_A(i) = 1$

Find $P_A(0)$ based on $P(0,0)$

Throughput Analysis: ALOHA with NC (cont.)

- Now we have $P_A(0)$, $P_B(0)$ and $P(0,0)$, throughput $\boldsymbol{S}$ can be calculated:

$$S = \mu P_{NT} + 2\mu P_{CT}$$

$$\begin{cases} P_{NT} = \left(P_A(0) - P(0,0)\right) + \left(P_B(0) - P(0,0)\right) = P_A(0) + P_B(0) - 2P(0,0) \\ P_{CT} = 1 - P(0,0) - P_{NT} = 1 - P_A(0) - P_B(0) + P(0,0) \end{cases}$$

- Final result (approximation because of $P(0,0)$)

$$S = \frac{2qG(2-G)}{q(2+G)^2 - G^2}, \quad \text{where } G = 2g$$

- Stabilization condition: $\lambda < \mu$ i.e., $q > \dfrac{G}{2+G}$

Throughput Analysis: ALOHA with NC (cont.)

- Maximization of throughput?

$$\frac{\partial}{\partial q} S = \frac{-2G^3(2-G)}{\left(q(2+G)^2 - G^2\right)^2} < 0, \quad \forall 0 < G < 2$$

- S monotonically decreases as q increases from $G/(2+G)$ to 1

  → maximized when $q \rightarrow G/(2+G)$

- What is P_{CT} in that case?

$$\lim P_{\mathrm{CT}} = 1 \;\; \text{when} \;\; q \rightarrow \frac{G}{2+G}$$

→ S is maximized thanks to considerable chance of NC

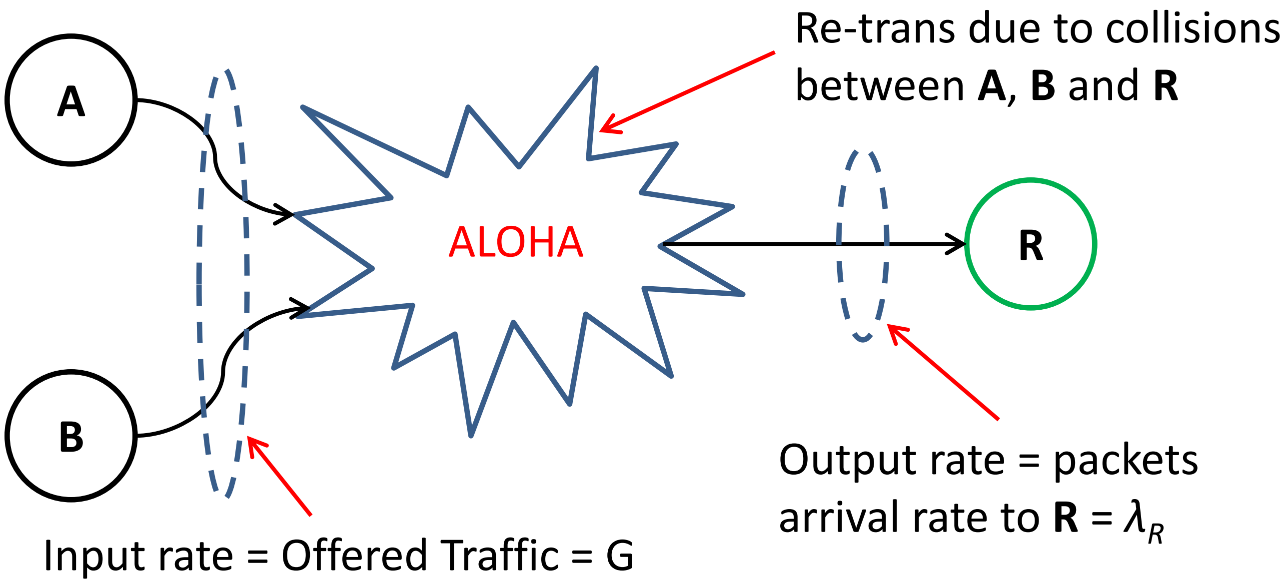
Throughput Analysis: ALOHA with NC (cont.)

- Throughput **S** has been derived
- How about average delay (D) per packet?
  1. Packet transmission & re-transmission delay from **A** & **B** until the packet reaches **R** (denoted by D_1)
  2. Delay at node **R**: include queuing, transmission and re-transmission delay until the packet reaches either **A** or **B** (denoted by D_2)

$$\rightarrow \; D = D_1 + D_2$$

Throughput Analysis: ALOHA with NC (cont.)

- 1^{st} type of delay (D_1)



- Avg. total number of transmissions (incl. 1^{st} trans + re-trans) until a packet reaches R : $N_T = \frac{G}{\lambda_R}$

Throughput Analysis: ALOHA with NC (cont.)

- If q satisfies stabilization condition, **R** is at equilibrium : arrival rate to **R** = departure rate from **R** = throughput **S**

$$N_T = \frac{G}{\lambda_R} = \frac{G}{S}$$

- Out of N_T transmissions :
  - 1 is for initial transmission
  - $(N_T - 1)$ are for re-transmissions. But a re-trans happen with prob. g_r in each slot → takes $1/g_r$ slots for a re-trans

$$\rightarrow D_1 = 1 + \frac{1}{g_r}\left(N_T - 1\right) = 1 + \frac{qG\left(6+G\right) - G^2}{2g_r q\left(2-G\right)}$$

Throughput Analysis: ALOHA with NC (cont.)

- 2nd type of delay (D_2) : using Little's theorem

$$D_2 = \frac{N_R}{\lambda_R}$$

  - N_R : average number of packets in **R**'s physical buffer

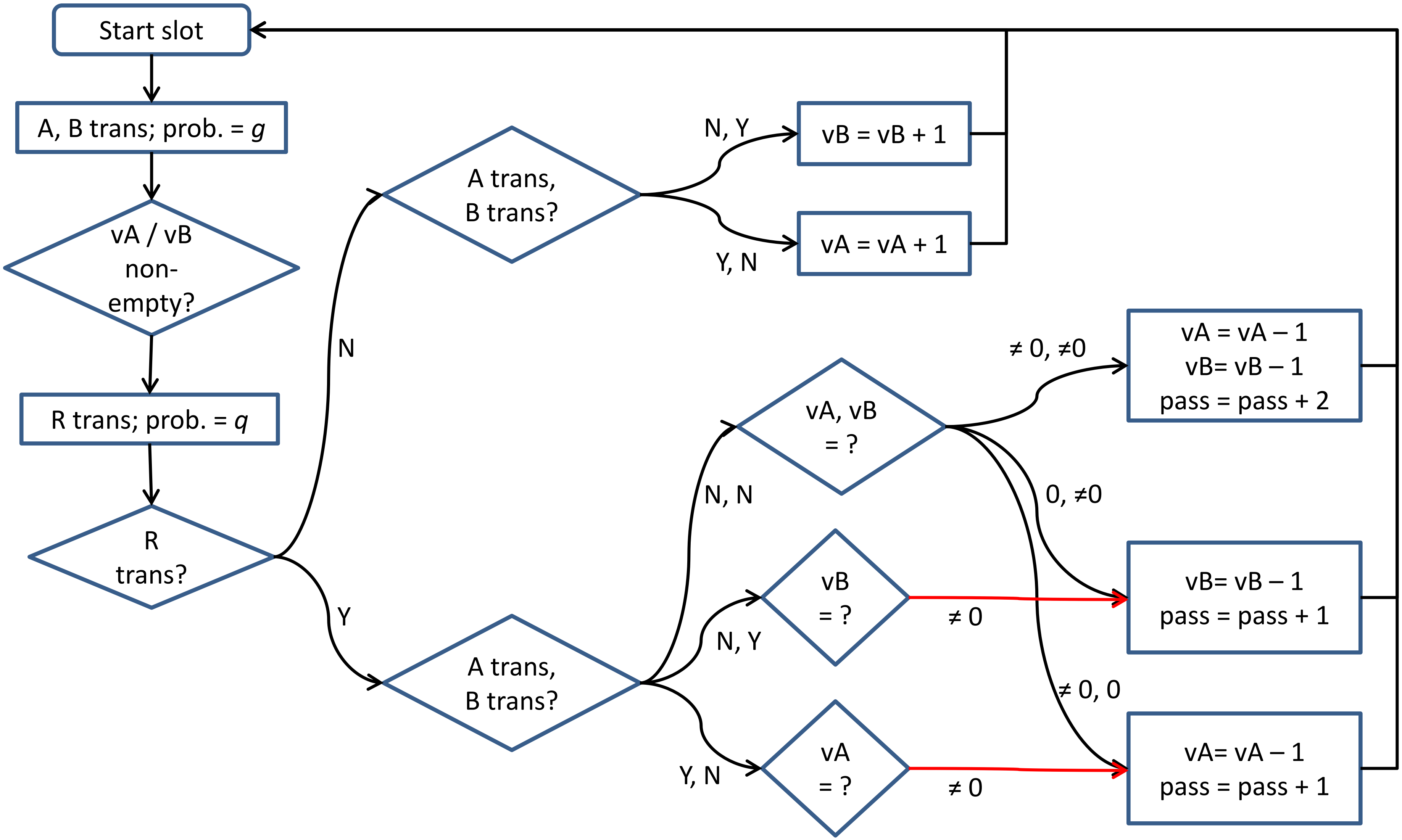
$$N_R = \sum_{\forall n} nP_A(n) + \sum_{\forall m} mP_B(m)$$

  - $\lambda_R = \boldsymbol{S}$ if q satisfies stabilization condition

- Final result

$$D_2 = \frac{4}{(2-G)\left(q(2+G)-G\right)}$$

Simulation results: NC-system flow diagram



Simulation results: NC-system